Economics Letters 25 (1987) 385–388
North-Holland

AN UPPER BOUND FOR WEALTH DISTRIBUTION IN A MODEL OF ACCIDENTAL BEQUESTS

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Received 25 June 1987
Accepted 23 September 1987

An explicit upper bound is calculated for wealth distribution in a model of overlapping generations with uncertain lifetimes and an imperfect annuities market

1. Introduction

In recent years, we have witnessed great interest among researchers in the models of uncertain lifetimes with incomplete annuities markets. For example, Abel (1985) has analyzed the implications of uncertain lifetimes on aggregate consumption and capital accumulation in a general equilibrium model where annuities are absent. Eckstein, Eichenbaum and Peled (1985) have shown how wealth distribution and welfare of different generations can be studied in an overlapping generations model in which lifetimes are uncertain.

This paper analyzes the effect of an incomplete annuities market on the distribution of wealth. We show that under some conditions frequently used in the literature, *the distribution of wealth is bounded above*. This result is important because in the literature it has been (most of the time, implicitly) asserted that imperfect annuities markets, even when individuals have no bequest motives, can account for 'a large part of aggregate wealth' [Abel (1985, p. 777)]. If this assertion holds, then the wealth inequality generated by these models must be large. In fact, Stiglitz (1978, p. 295) claimed precisely that '... an important source of inequality of inheritance is the stochastic nature of death'. He claimed further that the wealth distribution without any bequest motive will have a Pareto 'tail'. In the light of our finding of bounded wealth distribution, Stiglitz's assertion of a Pareto tail of the wealth distribution is incorrect.

The wealth distribution in the present context has been studied by Abel (1985) and Eckstein et al. (1985). Therefore, it is important to examine the difference between their approaches from ours. First, Eckstein et al. did not seek an upper bound for the wealth distribution. In contrast, Abel (1985, p. 781) did produce an upper bound for wealth distribution. However, his utility function was assumed to be of the hyperbolic absolute risk aversion type. In contrast, our utility function is assumed only to be concave. Second, Eckstein et al. assumed a one-for-one storage between time periods. In our model (and in this respect, in Abel's model as well), the presence of productive capital is permitted.

* I am indebted to Professor Andrew Abel for his comments on an earlier draft which led to the stronger result. However, I alone am responsible for all errors.

The rest of the paper is organized as follows. In section 2, we describe the model. In section 3, we present the result of bounded wealth distribution. Section 3 contains conclusions.

2. The model

Consider an economy of overlapping generations [Samuelson (1958)] populated with a continuum of individuals and a single commodity. If one unit of the commodity is invested in one period, it yields R units in the following period. At each time period t ($t \geqslant 1$), a new generation appears. Each newborn has a probability $1-p$ ($0<p<1$) of dying at the end of period 1 of her life. Before the resolution of this uncertainty, each member of each cohort gives birth to G offspring ($G \geqslant 1$). The offspring are identical to their parents and to each other in terms of their preferences. Each individual works during the first period of her life earning a fixed labor income w. An individual surely dies at the end of period 2 of her life. If the individual dies at the end of the first period of her life, her invested wealth belongs to her offspring. Each individual is assumed to maximize

$$u(c_1)+p\cdot bu(c_2), \tag{1}$$

where c_i represents the (possible) consumption of the individual in period i of her life ($i=1, 2$) and b is the time preference discount factor ($b \leqslant 1$). This utility function has been commonly used in the uncertain lifetimes literature. [1] We also make assumptions on the instantaneous utility function $u(.)$ to ensure interior solutions to the maximization problem described below. [2]

The budget constraints can be described easily by noting the assumption of excluded annuities markets:

$$c_1 \leqslant W-s, \tag{2a}$$

$$c_2 \leqslant sR, \tag{2b}$$

where W is the wealth of an individual including both bequest (if any) and earning in period 1 of her life and s is the period 1 saving. Note that there is no uncertainty in the aggregate because of our assumption of a continuum of individuals in each generation.

Before we proceed with the optimization problem, it should be noted that even if the economy starts with a set of identical individuals (in terms of inheritance), there will be intra-cohort differences in wealth as time passes. In particular, the inheritance of an individual will depend on how many *consecutive* ancestors of that individual died at the end of period 1 of their lives. If an individual has t (consecutive) ancestors, each of whom died at the end of period 1, we call her a *type* t individual [following Abel (1985)]. Note that a type t individual occurs in the economy with frequency $p(1-p)^t$. Therefore, in the absence of stochastic death (i.e., for $p=1$), there will be no inter/intra-cohort variation in wealth.

3. Economies with bounded wealth distribution

An individual's problem in the first period of her life is to maximize (1) subject to (2a) and (2b). Given the assumptions on $u(.)$, the necessary and sufficient condition for this problem is given by

$$u'(W-s)=pbR\,u'(sR). \tag{3}$$

[1] For an early example, see Yaari (1965). More recent examples would include Barro and Friedman (1977), Abel (1985), Eckstein et al. (1985) and Sinha (1986).

[2] Specifically, we assume $u'(.)>0$, $u''(.)<0$, $u'(c)\to 0$ as $c\to\infty$ and $u'(c)\to\infty$ as $c\to 0$.

For an individual of type t, we shall write $W = w_t$ and $s = s_t$ in (3). Notice that an explicit solution from (3) cannot be obtained (in terms of the other parameters of the model) unless (a) some explicit functional form of u is assumed or (b) $pR = 1$. However, an inequality can be obtained without such assumptions. The only assumption we need is

Assumption 1. $pbR \leqslant 1$.

From $u'' < 0$ and Assumption 1,[3] it follows that

$$w_t \geqslant s_t(1+R). \tag{4}$$

To see the implications of this assumption, we use the following simple fact which connects a type t with a type $(t-1)$ individual:

$$w_t = w + s_{t-1}\, R/G. \tag{5}$$

By combining (4) and (5), we get

$$w_t - w_{t-1} \cdot k \leqslant w, \tag{6}$$

where $k = R/[(1+R)G]$. Since $R > 0$, it follows that $k < 1$. Therefore,

$$w_t \leqslant w/(1-k) \quad \text{for} \quad \text{all } t. \tag{7}$$

The main implication of (7) is that the wealth distribution in such an economy is bounded above. Note that Assumption 1 is implicit in Eckstein et al. (1985), as they assume $R \equiv 1$ and $G \equiv 1$. However, Abel's bound depends on a parameter in the utility function; hence his bound is not strictly comparable to ours.[4] In Stiglitz (1978), Assumption 1 is implicit. Therefore, (7) contradicts Stiglitz's claim of a Pareto tail of wealth distribution – a distribution with a finite upper bound can never have a Pareto tail.

4. Conclusion

In this paper, we have explicitly calculated an upper bound for wealth distribution in a general equilibrium model of selfish overlapping generations with uncertain lifetimes, which yields several models studied in the literature as special cases. The main conclusion is that these models of accidental bequests have very limited abiblity to account for wealth distributions which have infinitely long right tails. To put it differently, we have demonstrated that models of uncertain lifetimes with selfish preferences do not fully explain the right tail of the wealth distributions.

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[3] If $pRb = 1$, then by $u'' < 0$, we can write $s_t = w_t/(1+R)$.
[4] Abel's condition for bounds are discussed in his footnote 11 on p. 781.

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