$$\begin{pmatrix} 0 & 0 & 0 & -1 & \gamma_1 & 0 \\ 0 & -\omega & -(1-\omega) & 0 & (1-\alpha) & 0 \end{pmatrix} \begin{pmatrix} r^* \\ d-w \\ f-w \\ g \\ e-w \\ v-w \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \alpha\gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{r}^* \\ \dot{d}-\dot{w} \\ \dot{f}-\dot{w} \\ \dot{g} \\ \dot{e}-\dot{w} \\ \dot{v}-\dot{w} \end{pmatrix} + \begin{pmatrix} 1 & \gamma_2 \\ 1 & -\lambda \end{pmatrix} \begin{pmatrix} y \\ i \end{pmatrix} + \begin{pmatrix} 0 & 0 & -\gamma_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{z} \\ \eta \\ \dot{e} \end{pmatrix} = 0 \quad \text{(C.2)}$$

MACROECONOMETRIC MODELING OF SAVING AND INVESTMENT FOR MERCOSUR COUNTRIES

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Resumen: En el largo plazo, el valor actual del balance de la cuenta corriente no puede ser indefinidamente grande sin caer en una crisis macroeconómica. Este hecho genera una hipótesis econométrica entre el ahorro y la inversión. Utilizamos datos de cuatro países del MERCOSUR: Argentina, Brasil, Paraguay y Uruguay. Los resultados indican que no hubo relación entre el ahorro y la inversión en estos países en el largo plazo durante 1950-1992. Así, es probable que MERCOSUR actue como paliativo contra tal posibilidad en el futuro.

Abstract: In the long run, the present value of current account balance can not grow indefinitely large without precipitating in a macroeconomic crisis. This simple insight produces an econometrically testable relationship between saving and investment. We use data for four countries, which belong to the MERCOSUR Common Trade Agreement: Argentina, Brazil, Paraguay and Uruguay. The results indicate that there is no long run relationship between saving and investment in these countries. Thus, MERCOSUR is likely to act as a palliative against such a possibility in the future.

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1. Introduction

Following the seminal paper of Feldstein and Horioka (1980), economists have been studying the relationship between saving and investment very intensively. Economists have mainly studied the relationship in order to understand international capital mobility in the industrialized countries in a better fashion. However, data availability for developing countries and recent advances in econometrics have helped economists to understand the relationship in the developing countries as well.

In this paper, we extend the Keynesian model to a dynamic stochastic multi-country environment with each country as the basic element of analysis. Our model produces a clearly econometrically testable hypothesis: saving and investment should be cointegrated. We test this theory using four MERCOSUR countries: Argentina, Brazil, Paraguay and Uruguay. The MERCOSUR Common Trade Agreement was signed in 1991. The four countries are responsible for one third of total trade and 70 percent of GDP of Latin America. Moreover, trade between MERCOSUR amounted to US$17 billion in 1996 (The Economist Intelligence Unit, 1998)

2. A Dynamic Stochastic Model of Saving and Investment

Suppose that there are n countries in the world, each of them small enough not to affect the world interest rate (R) individually. We will use subscript t to denote time and subscript i to denote a country. We formulate a variant of the linearized version of the model proposed by Feldstein (1983) which is used by Coakley *et al.* (1995):

$$S_{it} = a_k + S_{it-1} + b_k R_t + e_{kit} \tag{1}$$

Equation (1) summarizes the stylized facts that saving (S) is a process with unit root and that saving at time t for country i (S_{it}) depends positively on (real) world interest rate (R) at time t.

$$I_{it} = a_1 + I_{it-1} - b_l R_t + e_{lit} \tag{2}$$

Equation (2) encapsulates the stylized facts that investment (I) is also a unit root process but has a negative relation with interest rate.

More generally,

$$I_{it} = a_l + I_{it-1} - b_l R_t + d(S_{it-1} - I_{it-1}) + e_{lit} \tag{2'}$$

The additional term $d(S_{it-1} - I_{it-1})$ is an error correction term to reflect a risk premium. The terms e_{kit} and e_{lit} are i.i.d. white noise processes and a_k, b_k, a_l, b_l and d are constants. Two equations (1) and (2'), or (1) and (2) as (2') becomes identical to (2) if $d = 0$, can be used to solve for R_t:

Subtracting (2') from (1), we get

$$S_{it} - I_{it} = a_k - a_l + S_{it-1} - I_{it-1} + (b_k - b_l)R_t - d(S_{it-1} - I_{it-1}) + e_{kit} - e_{lit} \tag{3}$$

If it is a closed system with n countries, then total saving in each period must be equal to the total investment in that period. This means $\Sigma S_{it} = \Sigma I_{it}$ summing over i.

Summing over i in equation (3), we get,

$$0 = n(a_k - a_l) + n(b_k - b_l)R_t + \Sigma(e_{kit} - e_{lit}) \tag{4}$$

Thus, assuming $b_k \neq b_l$, we solve for R_t from (4) by noting that differences of independence white noise processes still produce white noise (say, z_t):

$$\begin{aligned} R_t &= [a_l - a_k - \Sigma(e_{kit} - e_{lit})/n]/(b_k - b_l) \\ &= (a_l - a_k)/(b_k - b_l) - \Sigma(e_{kit} - e_{lit})/n(b_k - b_l) \\ &= R^* + z_t \end{aligned} \tag{5}$$

Note that we can break the right hand side of the equation into two components, component wich does not depend on t and a second component of white noise (because it is a linear combination of white noises). We call the first component R^* (and note that it does not depend

on time t) and the second component z_t. We can interpret R^* as a global "time independent" component of the interest rate.

Let $C_{it} = S_{it} - I_{it}$. Then, from equation (3) we get by substituting the expression (5),

$$C_{it} = C_{it-1} - d(C_{it-1}) + (b_k - b_l)\,[R^* + z_t] + (e_{kit} - e_{lit}) \qquad (6)$$

We can take expectations in (6) on both sides conditional on time $t-1$ to get

$$E_{t-1}(C_{it}) = C_{it-1} - d(C_{it-1}) + (b_l - b_k)R^* \qquad (7)$$

[Note that $E_{t-1}(z_t)$ and $E_{t-1}(e_{kit}) = E_{t-1}(e_{lit}) = 0$ as they are white noise processes]

For a country i, the present value of the conditional expectations of C_{it} must be bounded above:

$$E_{t-1}(\Sigma C_{it}/(1+R^*)^t) < \infty \qquad (8)$$

Given (7), (8) follows provided $(1-d)/(1+R^*) < 1$. In fact, we can show by simple algebra that

$$E_{t-1}(\Sigma C_{it}/(1+R^*)^t) = C_{it-1}(1-d)(1+R^*)/(R^*+d) \qquad (9)$$

provided $(1-d)/(1+R^*) < 1$.

This solvency condition for the country shows that in the long run, the only credible path of saving and investment is one in wich they are cointegrated. Otherwise (9) does not hold. Moreover, the cointegrating vector should be $(1, -1)$ because $C_{it} = S_{it} - I_{it}$ by definition. Any other relation would not be viable in the long run. Thus, the model actually produces a testable hypothesis. However, since our model rests on a number of restrictive assumptions (such as a closed system assumption and a small country assumption), the empirical results may not exactly produce a cointegrating vector $(1, -1)$.

There is another implication of the cointegration between saving and investment. From the national income accounting identity, we know that $S_t - I_t = X_t - M_t$, where X_t is the export and M_t is the import of the country at time t. Therefore, imports and exports should be cointegrated

in the same fashion as the saving and investment. Some implications of this relation between export and import have been explored in Husted (1992) using US data.

3. MERCOSUR Countries: A Historical Perspective

All four countries in MERCOSUR (Mercado del Cono Sur, which literally means the market of the Southern cone) have a history of political and economic turmoil. Only in the 1990s, led by full democratization and economic liberalization in Argentina, have economies settled down in both political and economic areas. However, our data do not cover much of this tranquil period.

3.1. *Argentina*

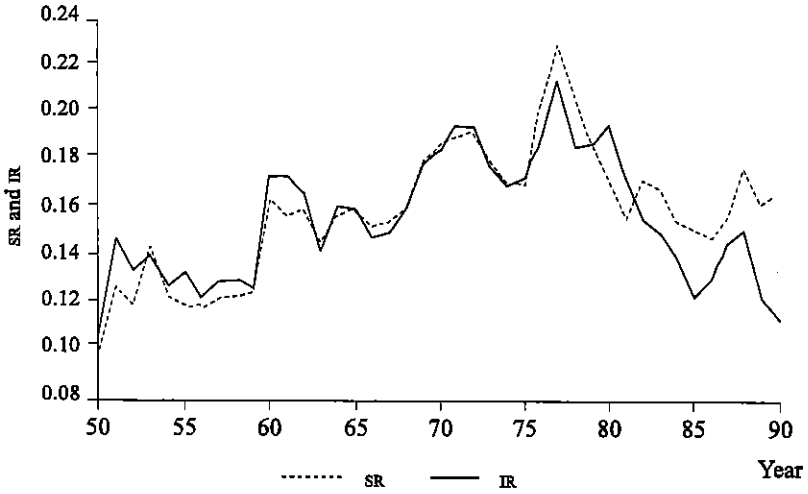
In the early 1950s, a fascist government led by Juan Perón won the support of powerful labor unions with the help of his colorful wife Eva Duarte de Perón. The spurt in saving and investment of the 1940s was not sustained as most of the resources were spent on popular but economically wasteful projects. The addition of several years of crop failures caused the economy to take a dip down. In 1958, Dr. Arturo Frondizi was elected the new president. He undertook austerity measures to stabilize the economy with the backing of United States. These measures produced a sharp rise in saving and investment. Unfortunately, this program was not popular and dissatisfaction led to a military-led coup in 1963.

For the rest of the decade, various military dictators ruled Argentina. The most important of this dictators was General Juan Carlos Ongania. He ran a repressive regime. The arbitrariness of military rule was not conducive to saving and investment. General Alejandro Lanusse launched a new program of land redistribution and economic growth. The economy responded with higher saving/investment and growth. The return of Juan Perón in 1973 and the subsequent ascendancy of his wife Isabel Martinez Perón to the presidency after his death produced a further, although short-lived, boost to saving and investment and for the first time in the post World War II, history of Argentina the investment rate fell below the saving rate. The main

reason for this fall in investment was that, because of removal of barriers to international investment, a substantial amount of money started flowing out of Argentina. Accelerated inflation in the 1980s brought more capital flight and several currency crises. As a result, investment fell sharply. This fact shows up in figure 1 as the investment rate dips below saving rate for the first time since 1950 for a prolonged period of time. The economy picked up in the 1990s, but our data end in 1990.

Figure 1

Saving Rate (SR) and Investment Rate (IR) for Argentina



3.2. *Brazil*

Brazil benefited substantially during World War II by siding with the Allies and supplying them with rubber and minerals. But General Getúlio Vargas who ascended to power in 1950 could not produce sustained growth in investment. After an initial spurt, the investment rate faltered and the economy was ravaged by inflation and other problems.

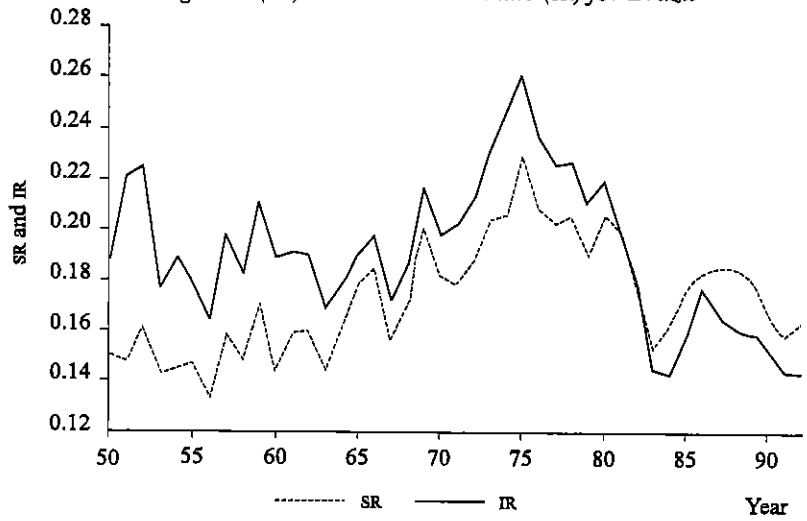
The growth of military had weakened the government. The economy suffered during the following decade. A commodity price boom in the late 1960s was the savior. This boom lasted for a decade. But the

rise in oil prices caused the demand for Brazil's commodities to fall, causing a substantial fall in commodity prices. This effect shows up in the halving of saving and investment between 1975 and 1982 (see figure 2).

Brazil practically defaulted on foreign loans. Rigid economic policies did not allow the economy to adjust to these external shocks. A fixed exchange rate led to the current account crisis and subsequent currency crisis. In 1990, the government of Collor de Mello devised an IMF sponsored plan of stabilization. Our data do not extend far enough to show the effects of this plan.

Figure 2

Saving Rate (SR) and Investment Rate (IR) for Brazil



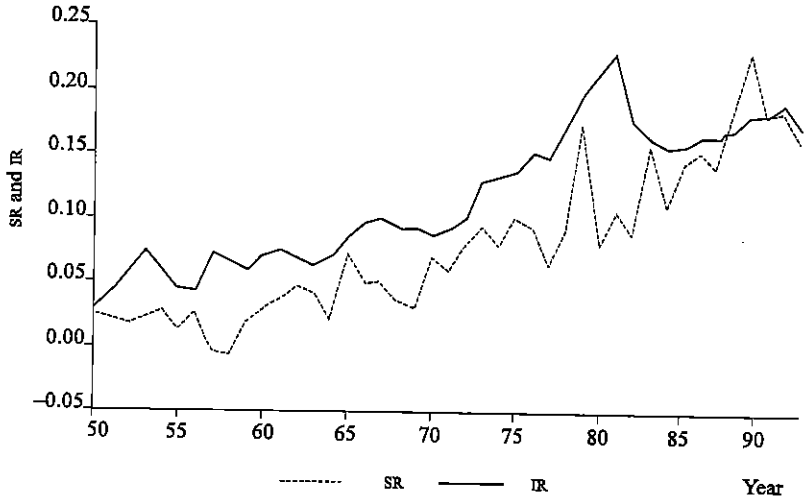
3.3. *Paraguay*

Only two South American countries are landlocked. Paraguay is one. The impact of the Triple Alliance War of the late last century and the Chaco War of the 1930s lasted well into the 1950s in Paraguay, as it lost more than half of its population in those wars. General Alfredo Stroessner ruled Paraguay with an iron fist from 1954 to 1989. His regime was politically

and economically very repressive. The means of production in all sectors of the economy were under government control. This led to widespread inefficiency and corruption. The single most important economic event in Paraguay took place between 1975 and 1982: the building of a dam in Itaípu. It was financed by American banks at a cost of $20 billion. However, the actual output from the hydroelectric project fell far short of the target. Paraguay defaulted on foreign loans, and in 1989 it had to renegotiate it loans with the "Paris Club". In 1989, Stroessner was overthrown in a coup. General Andrés Rodríguez assumed power and then held and won more or less free elections. During his presidency (1989-1993) he has moved to deregulate the heavily regulated economy. However data do not cover the recent time period to show the effects of deregulation (see figure 3).

Figure 3

Saving Rate (SR) and Investmet Rate (IR) for Paraguay



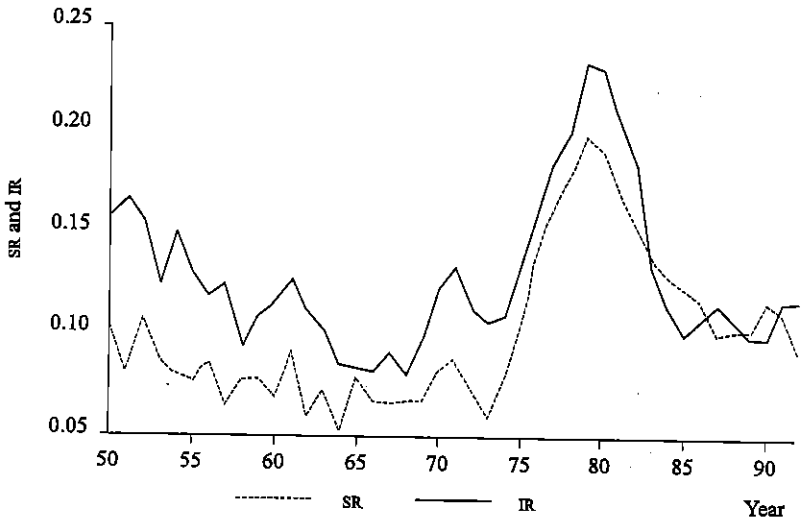
3.4. *Uruguay*

For Uruguay, the major economic activities revolve around livestock and agriculture. The major exports are meat, hide and wool. Uruguay adopted

the Swiss style multiple executive government in 1951, but it was a disaster. Moreover, the increased use of synthetic fiber in developed countries shrank the demand for wool and hide —the main exports of Uruguay. The decentralized government was burdened with a huge bureaucracy that drained the economy. Massive social unrest and insurgence followed throughout the 1960s. Not surprisingly, saving and investment suffered. However a huge surge in both variables took place in the late 1970s and early 1980s because of rising commodity prices and investment from international borrowing (see figure 4). A huge foreign debt resulted in the forced rescheduling of its debt in 1987.

Figure 4

Saving Rate (SR) and Investment Rate (IR) for Uruguay



Each of the four countries was characterized by a heavily regulated economy with repressive and corrupt military regimes at least for a part of the period under consideration. This is a recipe for a currency crisis. We would thus expect these countries to show signs of instability in the growth paths of saving and investment.

4. Future Prospects of MERCOSUR

When MERCOSUR was set up as a free trade agreement between Brazil and Argentina, and two smaller countries, Paraguay and Uruguay, it seemed too disparate a grouping. However, the deregulation of these economies (with a little help from the World Bank, International Monetary Fund and the "Paris Club") has led to faster integration than anybody imagined back in 1991 when the agreement was set up. In the following five years, trade between these countries has grown five folds (admittedly from a low base). In January 1995, its members agreed to form a customs union, setting a common external tariff averaging 14%. Chile has recently signed a free-trade agreement with MERCOSUR, and Bolivia and Venezuela are expected to follow soon. With Chile, MERCOSUR market will expand to include a total of 220m people with a combined GDP of nearly $1 trillion and total trade of $175 billion.

5. Tangible Dividends of MERCOSUR

MERCOSUR has already brought a number of political dividends. Paraguay's President Juan Carlos Wasmosy survived an attempted military coup in April of 1996 largely because of the support of his MERCOSUR partners. At their meeting in San Luis in June 1996, the presidents of the four partner countries formally agreed to make democracy a condition for membership. The trade agreement has also shown its economic worth. For example, increased exports to Brazil have helped Argentina to weather a recession. Argentine and Brazilian firms have formed more than 150 joint ventures. Chilean companies are already investing heavily in the MERCOSUR countries. MERCOSUR's leaders have wider ambitions: they have signed a framework agreement aimed at developing free trade with the European Union by 2005. Chile's pacific ports will be their routes to expanding trade with Asia.

6. Past Imperfect

Our paper deals with the past of the countries that form the core members of MERCOSUR. To see if MERCOSUR will succeed, we need to understand the implications of policies that these countries followed in the past. The stakes are higher for the two smaller countries: Paraguay and Uruguay. Any free trade agreement should benefit smaller countries more. Conversely, any hiccups in the system cost the smaller countries dearly. For example, in April 1996, Brazil changed its rules on textile imports. This change cost the Uruguayan exporters $40 million in six months. MERCOSUR members have followed very different economic policies in the past. Even with the new "openness", their monetary policies are not coordinated. Argentina pegs its currency to the US dollar but Brazil is gently devaluing. This sort of policy differential is not viable in the long run as European Monetary Union showed in the 1980s. Our paper highlights an aspect of the divergence in policies that may cause problems in the future for MERCOSUR.

7. What Can We Say About Cross Section Correlation between Saving and Investment?

Feldstein and Horioka (1980) upset conventional wisdom by proclaiming that a high saving-investment correlation in pooled cross-section data of a number of (industrialized) countries implies capital immobility among them. This assertion holds under very restrictive theoretical conditions, Frankel (1992). Moreover, simulations with artificial economies have shown that high saving and investment correlation can persist even with perfect capital mobility, Baxter and Crucini (1993) and Finn (1991). We deliberately refrain from drawing any conclusion based on pooled cross section analysis of our datasets for the following reason: all the basic series exhibit unit roots. Gonzalo (1994) has shown that in the presence of unit roots in the time series data, none of the usual test statistics for the ordinary least square regressions have standard distributions. Hence, any inference drawn from them are very likely to be erroneous even with very large samples. Therefore, applying their argument in these data series seems entirely inappropriate.

What can we say about the presence or the absence of cointegration between saving and investment? What we can say is the following: If capital is more mobile, then external shocks can be absorbed by the country (the absorption is manifested by the divergence between saving and investment).

8. Data and Methodology

All data come from the Penn World Table (version 5.6). Annual data are used as follows: Argentina (1950-1990), Brazil (1950-1992), Paraguay (1950-1992) and Uruguay (1950-1992). The two variables considered are gross domestic saving and investment as percentages of gross domestic product. We call these variables SR and IR respectively.

We use the Phillips-Perron (1988) unit root test. The test is well suited for analyzing time series whose differences may follow mixed ARMA (p, q) processes of unknown order because the test statistic incorporates a nonparametric allowance for serial correlation. Consider the following equation:

$$y_t = \tilde{c}_0 + \tilde{c}_1 y_{t-1} + \tilde{c}_2(t - T/2) + v_t \qquad (10)$$

where $\{y_t\}$ is the relevant time series in equation (10), T is the number of observations and v_t is the error term. The null hypothesis of a unit root is H_0: $\tilde{c}_1 = 1$. We can drop the trend term to test the stationarity of a variable without the trend.

The concept of cointegration is proposed by Granger (1981). Engle and Granger (1987) provide an axiomatic foundation of the methodology. Two (or more) $I(1)$ variables are said to be cointegrated if there exists a linear combination of them that is stationary. We use the Johansen-Juselius, see Johansen (1988) and Johansen and Juselius (1990) for details, tests for cointegration. The method can be shown to have the error correction representation of the VAR (p) model with Gaussian errors:

$$\Delta Z_t = a_0 + \Gamma_1 \Delta Z_{t-1} + \Gamma_2 \Delta Z_{t-2} + \ldots$$

$$\Gamma_{p-1} \Delta Z_{t-p+1} + \Pi Z_{t-p} + BX_t + u_t \qquad (11)$$

where Z_t is a an $m \times 1$ vector of $I(1)$ variables, X_t is an $s \times 1$ vector of $I(0)$ variables, Γ_1, Γ_2, Γ_{p-1}, Π are $m \times m$ matrices of unknown parameters, B is an $m \times s$ matrix and $u_t \sim N(0, \Sigma)$. The maximum likelihood method is used to estimate (11) subject to the hypothesis that Π has a reduced rank, $r < m$. The hypothesis, therefore, is as follows:

$$H(r)\colon \Pi = \alpha\beta' \qquad (12)$$

where α and β are $m \times r$ matrices. If certain conditions are fulfilled, equation (12) implies that the process ΔZ_t is stationary, Z_t is non-stationary, and that βZ_t is stationary. βZ_t is known as the cointegrating relation and β the cointegrating vector. In our model C_t plays the role of Z_t in (11). If we find that SR and IR are cointegrated, the relevant hypothesis for the vector β to be tested is H_0: $\beta' = (1, -1)$. Our results, however, have to be interpreted with caution. The unit root tests have low power, Blough (1992). The same is true for the Johansen-Juselius cointegration tests.

9. Results

The results of the Phillips-Perron unit root tests on the levels and first differences of the variables are in tables 1 and 2 respectively. For Paraguay, SR is found to have no unit root. For Argentina, Brazil and Uruguay countries, both variables are found to be $I(1)$. For Paraguay, IR is $I(1)$ but SR is $I(0)$. Therefore, we only perform cointegration tests for the remaining three countries. We use the finite sample correction proposed by Reinsel and Sung (1992). The trace statistic is multiplied by $T - p^k / T$ where T is the number of observations, p is the number of variables and k is the lag order in the VAR system. The results are in table 3. Although we do not report the results of the maximal eigenvalue tests, the results are the same with the maximal eigenvalue tests as well. Thus, our results are quite robust. The results indicate that SR and IR are not cointegrated for any of the three countries.

Table 1
Phillips-Perron Unit Root Tests for Levels of Saving Ratios and Investment Ratios

	SR		*IR*	
	Test Statistic	*Critical Value*	*Test Statistic*	*Critical Value*
Argentina	–2.2642	–3.5247	–2.2301[a]	–2.9358
Brazil	–2.4209	–3.5189	–2.1892	–3.5189
Paraguay	–5.2491	–3.5189	–2.1660	–3.5189
Uruguay	–2.1239	–3.5189	–2.2748[a]	–2.9320

[a] Indicates no trend.

Note: The critical values at the 5% level are from Mackinnon (1991). The lag of 3 was determined using the Schwert (1989) Criterion.

Table 2
Phillips-Perron Unit Root Tests for First Differences of Savings Ratios and Investment Ratios

	SR		*IR*	
	Test Statistic	*Critical Value*	*Test Statistic*	*Critical Value*
Argentina	–9.0971	–3.5279	–7.6800[a]	–2.9378
Brazil	–12.6241	–3.5217	–7.5276	–3.5217
Paraguay	NA[b]	NA[b]	–4.9706	–3.5217
Uruguay	–6.4419	–3.5217	–4.8107[a]	–2.9339

[a] Indicates no trend.
[b] Indicates Not Applicable (The variable does not have unit root in level form)
Note: The critical values at 5% level are from Mackinnon (1991). The lag of 3 was determined using the Schwert (1989) Criterion.

Table 3
Trace Tests for Cointegration between Saving and Investment

	Test Statistic	*Critical Value*
Argentina	7.6865	17.9530
Brazil	6.6588	17.9530
Paraguay	13.0757	17.9530
Uruguay	9.4873	17.9530

Note: The null and alternative hypotheses are $r = 0$ and $r > 1$ respectively. The critical value from Osterwald-Lenum (1992) is for the 95% quantile.

Thus, in all three countries, the ratios may drift apart. This clearly violates equation (9). Therefore, our results indicate that Argentina, Brazil, and Uruguay would be prone to sudden crises of currency or balance of payment problems resulting in macroeconomic adjustment problems such as high real interest rate or high inflation or both. Note that the result does not say anything about Paraguay because of the problem of mixing $I(1)$ and $I(0)$ variables. It does not, however, mean that Paraguay will not have such problems. The formation of MERCOSUR is likely to prevent such an occurrence for all of them. Thus, the trade agreement could not come at a more opportune time. As the four countries become more integrated, what will matter more is whether *the combined* SR and IR become cointegrated or not.

10. Conclusions

In this paper, we develop and test a variant of the Feldstein-Horioka hypothesis of saving-investment equality using the cointegration methodology. First, we test for unit roots. We find that except for the saving rate for Paraguay, saving and investment rates have unit roots for all four countries. This gives us a high degree of confidence on the equations (1) and (2) that incorporate the stylized facts. Tests show that both variables are $I(1)$ for Argentina, Brazil and Uruguay. Next, we proceed with the cointegration tests using the Johansen-Juselius framework. We use the finite sample correction to adjust our test statistics. The results show that saving and investment ratios do not have a long run relationship for any of the four countries. The divergence between saving rate and investment rate may result in macroeconomic instability in the long run. One of the potential benefits of the MERCOSUR agreement among these four countries is that such instability may become less likely. As the four countries become even more integrated, the combined saving and investment rates for these countries may have a positive long run relationship.

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ON CONSTANT ELASTICITIES OF DEMAND

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Consejo Superior de Investigaciones Científicas

Resumen: Mientras la matriz de Slutsky y la teoría de la dualidad se han utilizado para demostrar que las funciones de demanda con elasticidades constantes implican: elasticidades de renta unitarias, elasticidades cruzadas de la demanda nulas y elasticidades directas iguales a menos uno; esta nota demuestra que estos resultados se pueden obtener también directamente suponiendo simplemente que se verifica la ecuación de balance con estricta igualdad.

Abstract: While the Slutsky matrix and duality theory have been used to establish that constant elasticity demand functions imply unitary income elasticities, zero cross-price elasticities and own-price elasticities equal to minus one, this note shows that these results can also be strightforwardly derived from the simple assumption that demand functions satisfy the budget constraint with strict equality.

1. Introduction

The easy simplicity of the elasticity concept and the fact that elasticities are pure numbers have led economists to see their estimation as a primary aim of empirical studies. In econometric studies of consumer demand it has long been a common practice to assume that demand are elasticities constant. While this simple assumption facilitates the parametric estimation of demand elasticities (since it means that the underlying demand functions must be log-linear), it also imposes restrictions upon the magnitude of the elasticity values.[1] Slutsky equations, as well as duality

[1] Pareto (1911, pp. 613-616) was early in pointing out that the Marshallian constant demand curve is inadmissible, except in the special case when elasticity is unity.