MAKING OPTIMAL SPACE RENTING DECISIONS

Using mathematical programming, managers of multiple properties can achieve target while minimizing the rental cost.

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Large organization property managers routinely rent space. If a company is expanding its business in a particular city, it may have to set up new offices, new warehouses, and so on. Typically, managers make these choices sequentially. They lease or rent a space and place it in limbo. The next renting or leasing decision ignores the preceding decision.

This is a flawed approach. Whenever there is a choice, there is a decision to be made. These decisions will be better (cost will be minimized or some other objective achieved) if the manager views all the leased properties as a portfolio. The examples in this article will be limited to achieving target while minimizing the use of space.

A caveat about these decisions is in order. World peace depends on political decisions; individual happiness depends on personal decisions. Although these types of decisions are important, they are not quantifiable. Even fewer decisions are quantifiable in dollars and cents. This article deals only with leasing decisions for which objectives and constraints are quantifiable in dollars and cents. If, for example, the leasing decision depends on whether one can see the Sydney Harbour Bridge from the office in the Central Business District in Sydney, the manager cannot apply the methods discussed below. On the other hand, property managers who want to minimize the total cost of leasing subject to some quantifable constraints can use the method described.

To make correct or optimal decisions, decision makers have to identify the objectives. They also must identify the restrictions under which they have to fulfil the objectives. The identification of quantifiable restrictions and quantifiable objectives come under the general purview of "mathematical programming." (The use of "programming" in this regard did not arise from the term computer program, but from military operations. During World War II, attacks on the enemy were euphemistically referred to as "programs." The technique was first used successfully during WWII to schedule bombing raids on Germany. After the war, the applications spread quickly in the fields of business, economics, and chemical and industrial engineering and was incorporated into a specialized field called Operations Research (OR), a term that still reflects its military origin.)

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OPTIMIZING A DECISION UNDER CONSTRAINTS

Not all problems have solutions. If the constraints are incompatible with one another, no feasible solution might exist. But for practical problems, the constraints arise out of past experience. Hence, a carefully laid out problem is unlikely to have no feasible solution.

Typically, many possible solutions to a problem satisfy the constraints and are known as "feasible solutions." Among the feasible solutions, one solution is an "optimal solution," the solution that optimizes (maximizes or minimizes the objective function).

An Example

These concepts become clear with the help of the example illustrated in Exhibit 1. This problem, one of cost minimization in rental properties, is a simple problem with simple restrictions. (In the

EXHIBIT 1 A Problem of Cost Minimization in Rental Properties[a]

	Dollars per Square meters per Annum	Optimal Solution: Space Rented	Cost of Renting
Rent in Sydney	$350	8,000 sqm	$2,800,000
Rent in Brisbane	250	10,000	2,500,000
Rent in Melbourne	300	9,000	2,700,000
Total cost			$8,000,000

Minimum Space Requirement City by City

Sydney	5,000 sqm
Brisbane	2,000
Melbourne	4,000

Combined Space Requirements

S + M	17,000
S + B	18,000
B + M	19,000

[a] A budget of $8 million is given; some constraints on space requirements are also given.

jargon of mathematical programming, it is a linear programming problem!).

The problem arises because government departments have offices all over the country, each department requiring a minimum amount of space in a given city. Budgets usually also impose constraints on how much space a department may have in a given region.

A manager receives a budget of $8 million from the Department of Veterans Affairs (DVA) to rent space in three cities: Sydney, Melbourne, and Brisbane. The manager encounters the following constraints:

- **Price constraints (the rent).** In Sydney, the rent is $350 per square meter per annum, in Brisbane $250 and in Melbourne $300. In the jargon, these numbers are called the "parameters of the model."

As it stands, this problem is simple and uninteresting. The manager can save all the money by renting no space anywhere at all! Clearly, that is not what the DVA has in mind. Hence, the manager must identify more restrictions.

- **Linear restrictions.** The DVA also specifies that there must be at least 5000 square meters of space in Sydney, 4000 square meters in Melbourne and 2000 in Brisbane.

Unfortunately, dealing with these restrictions alone will not provide an interesting solution to the problem of minimizing costs. Why? The manger has no decisions to make. To minimize cost, one simply gets 5000 square meters in Sydney, 4000 in Melbourne and 2000 in Brisbane. In the language of a liner programming problem, this is a solution in which all the restrictions are binding.

To make the example interesting, it must include some other restrictions. Here are some simple ones: (1) Rentals in Sydney and Melbourne combined should be at least 17,000 square meters; (2) rentals of both Sydney and Brisbane should be at least 18,000 square meters of floor space; and (3) Melbourne and Brisbane together should have at least 19,000 square meters of space.

Admittedly, this problem is somewhat artificial. But not entirely, because the DVA serves many cities around Australia. Each Australian state might demand certain minimum service. Such restrictions can be translated into a minimum floor space requirement within each state. The example could have considered many other restrictions. For example, energy costs differ in various cities and states. So do cleaning costs and other building expenses. Any quantifiable restriction can readily be incorporated into the problem.

Feasible Solutions

The problem as it is now set up is not a very simple one. There is no obvious optimal solution. To state the problem as one in linear programming, let S be the amount of floor rented in Sydney, M, the amount in Melbourne and B, the amount in Brisbane. Then, the total cost of renting (the objective function of the problem) can be written as

$$250B + 300M + 350S$$

EXHIBIT 2 Solving A Rent-Minimizing Problem Constraints

Name of Constraint	Final solution	Requirement	Status
S+M sqm per annum	16,000	>=16,000	Binding
S+B sqm per annum	18,000	>=18,000	Binding
B+M sqm per annum	19,000	>=19,000	Binding
Sydney space rented	7,500	>=5,000	Not binding
Brisbane space rented	10,500	>=2,000	Not binding
Melbourne space rented	8,500	>=4000	Not binding

Solutions for Each City

Location	Initial choice	Final choice
Sydney space rented	8,000	7,500
Melbourne space rented	9,000	8,500
Brisbane space rented	10,000	10,500
	Original Budget	Final Cost
Total cost of Renting	$8,000,000	$7,800.000

The problem is to minimize this function.

The constraints are:

$M \geq 4{,}000$, $B \geq 2{,}000$, and $S \geq 5{,}000$

$S + M \geq 16{,}000$, $S + B \geq 18{,}000$, and $M + B \geq 19{,}000$.

Is there any feasible solution to the problem? The answer is yes. S = 8,000, M = 9,000 and B = 10,000 satisfy all the constraints. If a feasible solution is such that one of the restrictions holds with equality, it is called a "binding restriction." Since S + B = 18,000, it is a binding constraint for this solution. However, M > 4,000. Hence, this constraint is not binding. Is the solution within the budget? For this solution, the total cost of renting is $8 million exactly, and within the budget limit. Is this the best that can be achieved? In other words, is it possible to actually stay within the budget and satisfy the constraints and have the total operation cost less than $8 million? Again, the answer is yes.

The Optimal Solution

Finding a better solution is one thing. However, linear programming does something better. It finds the best solution, the optimal solution, the solution that minimizes the total cost. To put it differently, there will be no other solution that is any better!

Exhibit 2 concludes that $S^* = 7{,}500$, $M^* = 8{,}500$ and $B^* = 10{,}500$ is a better solution than the first. It satisfies all the constraints yet, at the same time, it reduces the cost to $7.8 million. There is no solution that satisfies all the constraints and achieves a lower cost.

Ease of Computation

The manager never has to calculate the solution. It is determined by a computer program. However, it is instructive to know how one actually finds such a solution. The linear nature of the problem ensures that the computer need make only a finite search. It looks at all the feasible solutions and finds the solution with the optimal value of the objective function.

Until just a few years ago, finding solutions to mathematical programming problems required the use of large mainframe computers. But today, there are many programs available on personal computers for solving such problems. The example in this article was solved using "Solver" in Microsoft Excel (Version 4). But other well-known packages can also be used.

Extension of the Model

Very rarely does the manager gets a chance to allocate space across different properties from a "clean slate." At a given moment there are existing leases. The manager's decision usually is to add to the existing portfolio of properties. Can linear programming solve for situations in which cash flows are taking place at different points in time? The answer is affirmative. A sequence of cash flows over time can be summarized by the present value of those cash flows given a discount rate. So, it is possible to collapse the problem arising from a series of payments over different time periods into a single period problem using some discount rate. Alternatively, if the problem is inherently intertemporal, the manager can produce an optimizing problem to take that into account. So, for example, if the problem has a constraint that says 8,000 square meters of space must be obtained in Sydney in 1993 (because the existing lease does not allow any less), that condition can be incorporated as a constraint, and a problem set up where several functions are maximized simultaneously (one for each time period). ■