

OPTIMAL PATHS TO RETIREMENT

By

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Abstract:

Mexico has adopted a newly privatized pension system for all workers in the formal sector of the economy. There are 14 retirement funds (AFORES) to choose from. Every worker is allowed to switch from one AFORE to another every year. What is the best path to choose if the worker tries to minimize service charges over lifetime? Charges imposed by the AFORES vary a great deal. Some of funds charge according to the flow of money. Others charge according to the account balance and yet others charge on the rate of return. Some even charge on a combination of some or all of the factors mentioned. The only way to work out the optimal path is to consider all the possible strategies with stochastic interest rates that are correlated over time. In this paper, we propose a method of working out the optimal path to retirement. The results show that under most possible scenarios there is no single best AFORE. However, the optimal path typically does not involve switching every year. Under most plausible assumptions, the optimal path involves two or three switches over the lifetime (30 to 40 years) of a full time worker.

INTRODUCTION

On July 1, 1997, a brand new privatized scheme of pension came into existence in Mexico. Each fund in the new system is called Administradora de Fondos de Retiros or an AFORE. In early March 1997, 17 AFORES were authorized by the regulatory body called CONSAR. The AFORES in Mexico are similar to the AFPs in Chile. They are private, most of them are subsidiaries of banks and insurance companies in Mexico and so far, highly regulated in what kind of market instrument they can invest in. Since its inception, it has produced quite a number of surprising results:

1. Number of affiliates to the system has grown rapidly:

Below we reproduce (Table 1) the number of affiliates with each AFORE. Almost 14 million workers have already signed up for some AFORE. This includes about 10 million active contributors (others 4 million are currently not contributing such as unemployed, recently retired and so on). The total number of workers in the formal sector is slightly under 15 million. Thus, what we have represents a remarkably swift move by the workers to be affiliated with the system. In this sense, the system has really achieved in a very short time what it was supposed to do. Many observers were skeptical about the spread of the new system. The speed was a surprise.

Table 1

AFORE	end of 1998 number of affiliates
Atlántico Promex	188,205
Banamex Aegon	1,568,595
Bancomer	2,226,239
Bancrecer Dresdner	607,522
Bital	1,304,719
Capitaliza	52,998
Confía Principal	114,340
Garante	1,533,250
Génesis	141,542

Inbursa	316,909
Previnter	*
Profuturo GNP	1,929,819
Santander Mexicano	1,968,585
Sólida Banorte Generali	1,190,605
Tepeyac	141,282
XXI	423,813
Zurich	119,251
System total	13,827,674
*Previnter numbers are shown with Profuturo	

Source: CONSAR

2. The amount of money invested:

The amount of money in the system has also increased steadily. In the first year of existence (July 1997 and July 1998), the investment was about US\$3 billion (at an exchange rate of 10 pesos per US dollar). Over the next seven months (July 1998 and January 1999), another US\$3 billion was invested. If this trend continues, in 25 years, AFOREs will have 40% of GDP in the system.

3. The investment portfolio:

CONSAR (the regulatory body of AFOREs) has stipulated that a minimum of 51% of investment has to be in inflation indexed bonds and at least 65% in assets with maturity no more than 183 days. At present (January 31, 1999) over 75% investment is in inflation indexed bonds (called BONDE91 and UDIBONOS). Another 15% are in CETES (Mexican Treasury Bills). The average maturity of the investment portfolio of the system is 111 days, well below the stipulated 183 days.

4. Performance of the investment portfolios:

Because of severe restrictions on the portfolios, there does not seem to be much variation in the rate of return of the funds. The best performing fund in real terms over the 19 months has been Profuturo GNP with 8.16% and Tepeyac the worst with 5.79% with system-wide average of 6.96%. Is that good? If there were no charges in any of these funds, it would be very good. Unfortunately, charges account for 20%-25% of every peso deposited. Thus,

the rate of return after counting charges is negative. One argument used by funds to justify high charges is that they manage funds to maximize the returns for the affiliates. In that case, the funds should perform much better than a simple portfolio where money is invested simply in BONDE91 and CETES that meet the minimum statutory requirements. In such a fund, the rate of return would have been 6.20%. This fund would be passively managed and hence free from commissions.

5. Relation between the number of affiliates and cost of advertising:

Funds generally argue that much of their commission they charge go into advertisement and other publicity to attract more customers. I have gathered some data on publicity cost of AFOREs and the number of affiliates they have. There must be a very strong relation between publicity cost and the number of affiliates each fund has. Unfortunately that does not hold. To put it differently, funds are wasting a lot of money in advertising that is not producing any results.

6. "Churning" or switching funds:

One of the main features of the Chilean privatized pension system is the number of people switching funds: approximately one in three change funds every year. This imposes a huge cost on the system. In case of Mexico, there is very little evidence that people are switching funds. In the past six months (since they were allowed to switch funds) less than 200,000 affiliates have switched funds. For the 10,000,000 active contributors this represents a very small proportion indeed.

How does a worker assess the performance of a fund? With a less than two year experience, there is not much to compare except in terms of charges of the funds. In this paper, we concentrate on the issue of charges. Specifically, we ask the following question: How does one compare charges among different funds? How does one choose an optimal strategy to determine if they should change funds mid-stream?

THE PROBLEM

The problem can be seen easily when we examine the fee structure of various funds. This is illustrated in the following table.

Table 1: Fee structure of AFOREs

AFOREs	Charges on flow each year (% of wages)	Charge on account balance	Charge on real rate of return
Atlantico Promex	1.40%		20.00%
Banamex	0.002 in 1997 0.85% in January 1998 1.70% in March 1998 onward		
Bancomer	1.70%		
Bancrecer Dresdner	1.60%		
Banorte	1.00%	1.50%	
Bital	1.68%		
Capitaliza	1.60%		
Confia Principal	0.90%	1.00%	
Garante	1.68%		
Genesis	1.65%		
Inbursa			33.00%
Previnter	1.55%		
Profuturo GNP	1.70%	0.50%	
Santander	1.70%	1.00%	
XXI	1.50%	0.99%	
Tepeyac	1.17%	1.00%	
Zurich	0.95%	1.25%	

How do we compare Zurich with Tepeyac? From the Table it is not at all obvious how this can be done. In addition, some funds charge different amounts depending on how long a person has been with the fund.

We start with 14 funds. However, given that a person can switch fund without penalty every year, over a period of 40 years there are $7'000'376'965'910.7 \times 10^{35}$ possible choices. It is virtually impossible to examine all the possible time paths. We, therefore restrict our attention only to a possible switch every five years instead of every year. This assumption

reduces the dimensionality of the problem considerably. In addition, examination of fee structure also reduces choices as some funds very clearly dominate others.

Consider the following Table.

Yr	Baname	Bancome	Bancrece	Bitá	Garant	Génesi	min
1	1.70%	1.68%	1.60%	1.68%	1.68%	1.65%	1.60% Bancrecer
2	1.70%	1.67%	1.60%	1.68%	1.68%	1.65%	1.60% Bancrecer
3	1.70%	1.66%	1.60%	1.68%	1.68%	1.65%	1.60% Bancrecer
4	1.70%	1.65%	1.60%	1.68%	1.68%	1.65%	1.60% Bancrecer
5	1.70%	1.64%	1.60%	1.68%	1.68%	1.65%	1.60% Bancrecer
6	1.68%	1.63%	1.58%	1.66%	1.68%	1.65%	1.58% Bancrecer
7	1.66%	1.63%	1.56%	1.64%	1.68%	1.65%	1.56% Bancrecer
8	1.64%	1.63%	1.54%	1.62%	1.68%	1.65%	1.54% Bancrecer
9	1.62%	1.63%	1.52%	1.60%	1.68%	1.65%	1.52% Bancrecer
10	1.60%	1.63%	1.50%	1.58%	1.68%	1.65%	1.50% Bancrecer
11	1.58%	1.63%	1.48%	1.58%	1.68%	1.65%	1.48% Bancrecer
12	1.56%	1.63%	1.46%	1.58%	1.68%	1.65%	1.46% Bancrecer
13	1.54%	1.63%	1.44%	1.58%	1.68%	1.65%	1.44% Bancrecer
14	1.52%	1.63%	1.42%	1.58%	1.68%	1.65%	1.42% Bancrecer
15	1.50%	1.63%	1.40%	1.58%	1.68%	1.65%	1.40% Bancrecer
16	1.48%	1.63%	1.38%	1.58%	1.68%	1.65%	1.38% Bancrecer
17	1.46%	1.63%	1.36%	1.58%	1.68%	1.65%	1.36% Bancrecer
18	1.44%	1.63%	1.34%	1.58%	1.68%	1.65%	1.34% Bancrecer
19	1.42%	1.63%	1.32%	1.58%	1.68%	1.65%	1.32% Bancrecer
20	1.40%	1.63%	1.30%	1.58%	1.68%	1.65%	1.30% Bancrecer
21	1.38%	1.63%	1.28%	1.58%	1.68%	1.65%	1.28% Bancrecer
22	1.36%	1.63%	1.26%	1.58%	1.68%	1.65%	1.26% Bancrecer
23	1.34%	1.63%	1.24%	1.58%	1.68%	1.65%	1.24% Bancrecer
24	1.32%	1.63%	1.22%	1.58%	1.68%	1.65%	1.22% Bancrecer
25	1.30%	1.63%	1.20%	1.58%	1.68%	1.65%	1.20% Bancrecer
26	1.28%	1.63%	1.18%	1.58%	1.68%	1.65%	1.18% Bancrecer
27	1.26%	1.63%	1.16%	1.58%	1.68%	1.65%	1.16% Bancrecer
28	1.24%	1.63%	1.14%	1.58%	1.68%	1.65%	1.14% Bancrecer
29	1.22%	1.63%	1.12%	1.58%	1.68%	1.65%	1.12% Bancrecer
30	1.20%	1.63%	1.10%	1.58%	1.68%	1.65%	1.10% Bancrecer
31	1.18%	1.63%	1.08%	1.58%	1.68%	1.65%	1.08% Bancrecer
32	1.16%	1.63%	1.06%	1.58%	1.68%	1.65%	1.06% Bancrecer
33	1.14%	1.63%	1.04%	1.58%	1.68%	1.65%	1.04% Bancrecer
34	1.12%	1.63%	1.02%	1.58%	1.68%	1.65%	1.02% Bancrecer
35	1.10%	1.63%	1.00%	1.58%	1.68%	1.65%	1.00% Bancrecer
36	1.08%	1.63%	0.98%	1.58%	1.68%	1.65%	0.98% Bancrecer
37	1.06%	1.63%	0.96%	1.58%	1.68%	1.65%	0.96% Bancrecer
38	1.04%	1.63%	0.94%	1.58%	1.68%	1.65%	0.94% Bancrecer
39	1.02%	1.63%	0.92%	1.58%	1.68%	1.65%	0.92% Bancrecer
40	1.00%	1.63%	0.90%	1.58%	1.68%	1.65%	0.90% Bancrecer

This table clearly shows that among these funds, Bancrecer charges the minimum regardless of years in the fund. Therefore, we can simply consider Bancrecer and ignore the others except when a person switches funds after a number of years.

Table 4

Group 2	Commissions on	
	Flow	Balance
2.1 Principal *	0.90%	1.00%
2.1 Santander Mex.	1.70%	1.00%
2.1 Tepeyac	1.17%	1.00%
2.2 XXI	1.50%	0.20%
2.2 Profuturo GNP	1.70%	0.50%
2.3 Zurich	0.95%	Variable
2.3 Banorte	1.00%	Variable

* It has a discounts scheme

From this Table, we can see that there are three comparable sub-groups of AFOREs in terms of their commissions. In the first subgroup, we have the following AFOREs: Principal, Santander Mexicano and Tepeyac. They all charge 1% on the balance but Principal charges less than the other two on the flow of funds. Thus, it is sufficient to consider Principal from this subgroup. then from this sub-group we only consider Principal for the analysis. In the second subgroup is XXI and Profuturo where the last one charges more than the first one in both cases (balance and flow) therefore, it is sufficient to only consider XXI and not Profuturo. Finally, the last subgroup consists of Zurich and Banorte and Zurich charges consistently below Banorte. We reduce our list to five AFOREs.

Table 5

Structure of Commissions			
		Flow	Return
Banamex	*	1.70%	balance
Bancomer	*	1.68%	
Bancrecer	*	1.60%	Variable
Banorte		1.00%	
Bital	*	1.68%	1.00%
Confía	*	0.90%	
Garante		1.68%	33.00%
Génesis		1.65%	
Inbursa			
Profuturo GNP		1.70%	0.50%
Santander Mex.		1.70%	1.00%
Tepeyac		1.17%	1.00%
XXI		1.50%	0.20%
Zurich		0.95%	Variable

		Flow	balanc	Return
Bancrecer	*	1.60%	1.00%	33.00%
Confía	*	0.90%		
Inbursa				
XXI		1.50%	0.20%	
Zurich		0.95%	Variable	

*discount for length of stay

However, it is still not sufficient to compare all the funds. The reason is the following: in addition to their own contribution, each affiliate of an AFORE gets a contribution from the government (called the cuota social). This amount is indexed and the value has been set at 5.5% of the minimum salary of Mexico City as of July 1, 1997. Some funds charge on the nominal balance (in addition to charges on contribution). Thus, there is a mismatch in how charges apply to different parts of the fund balance. Moreover, if somebody's monthly income is close to the minimum salary, the balance in his account will have a far greater impact from the cuota social than a person whose salary is ten times the minimum salary. To put it differently, the presence of cuota social puts a severe nonlinearity into the system making it difficult to compare charges over a number of years. In the paper, we resort to a number of simulations to get a general feel for the comparability of funds.

DEVELOPING THE MODEL

Essentially, individual retirement benefits are calculated by using a future value formula. However, the simple future value formulas we find in Kellison (1991) or other similar treatment does not deal with some of the complexities we find in the Mexican system: (1) Government contribution to the individual account does not apply every month, and the indexing is also not applicable monthly. (2) Commissions come in three basic flavors (a) commission over the flow of funds, (b) commission over the account balance and (c) commission over the *real* rate of return. In addition, some companies charge commission by combining (a), (b) and (c) (3) In addition, the commissions mentioned in (2) do not stay constant over time. They vary with the number of years one stays in the fund. (4) Income of each individual does not stay constant during his/her working life. Such changes have to be taken into account. For these reasons, the following discussion will be based on a recursive development of the formula for calculating retirement benefits.

WHAT IS THE RIGHT MEASURE OF COST?

Because charges apply to different parts of the AFORE, it is not easy to compare charges across AFORES. If we look at the system as a whole, there is a problem of charges when the system starts up. Charges appear too high! In Chile, for example, in 1984, charges amounted to 9% of wages or 90% of contributions to the retirement system (Edwards (1996), p. 17). However, the costs have come down to about 15% of contributions in 1990, (see, World Bank, (1994), p. 224).

For individual AFORES, it makes it difficult to compare across funds. For example, suppose we want to compare the charges for Inbursa and Banamex. Since Banamex charges 26.15% of total contribution up-front but Inbursa charges nothing up front, it may seem like charges for the AFORE run by Banamex is very high. However, charges for Inbursa are complicated because their charges apply to the *real rate of return*, over the long run, it adds up. Thus, it makes little sense to calculate charges as a percentage of total assets in a system that just starts up.

There are several ways to look at the charges: (1) operating costs as a percentage of total annual contribution, (2) operating costs as a percentage of average total assets, (3) operating costs as a percentage of covered annual wages, (4) operating costs as a percentage of affiliates times per capita income.

There are two components of the new system: (1) contribution by the worker, (2) contribution by the government. The contribution by the worker is 6.5% of his or her base wage. The contribution by the government is 5.5% of the minimum salary *indexed to the rate of inflation*. There are two additional complications: (1) interest rate is calculated for

every account every *two* months and (2) indexation of the government contribution takes place every *three* months.

Therefore, we can write the accumulated value in the AFORE as follows in a recursive formula in the simplest case:

$$S_k = \begin{cases} (6.5\% * BW * 2 + GC) * (1 + i_1^{(12)}) & k = 1 \\ S_{k-1} * (1 + i_k^{(12)}) & k = 2i \quad i = 1, 2, \dots, \frac{CP}{2} \\ (S_{k-1} + (6.5\% * BW * 2 + GC)) * (1 + i_k^{(12)}) & k = 2i + 1 \quad i = 1, 2, \dots, \frac{CP - 2}{2} \end{cases}$$

where, the government contribution G (cuota social), can be written as follows:

$$GC = \begin{cases} SocC = 5.5\% * MW & k = 1 \\ SocC = SocC * (1 + \pi^{(4)}) & k = 3i \quad i = 1, 2, \dots \end{cases}$$

where MW is the minimum wage and $i_k^{(12)}$ is the nominal interest rate and CP is the contribution period.

There is one peculiar aspect of the formula above: calculation of benefit account uses a *simple interest* rate for the adjustment for one month's rate of return to a bimonthly rate. Therefore, we get the factor BW.2 in the above equation.

Some AFORES have charges on contribution as a percentage of wages (for example, for Banamex). Others have charges on the balance in the AFORE account (such

as Bancrecer). Still others have charges on the real interest rate (such as Inbursa). Let CW be the charge on wage (rate). [See Table 2]

CHANGING THE FORMULA: CHARGES

Let CB be the charge on balance. We need to modify the above formula as follows:

$$S_k = \begin{cases} \left(6.5\% * BW * 2 * \left(1 - \frac{CW}{6.5\%} \right) + GC \right) * (1 + i_1^{(12)}) * \left(1 - \frac{CB}{12} \right) & k = 1 \\ S_{k-1} * (1 + i^{(12)}) & k = 2i \quad i = 1, 2, \dots, \frac{CP}{2} \\ \left(S_{k-1} + \left(6.5\% * BW * 2 * \left(1 - \frac{CW}{6.5\%} \right) + GC \right) \right) * (1 + i_k^{(12)}) * \left(1 - \frac{CB}{12} \right) & k = 2i + 1 \quad i = 1, 2, \dots, \frac{CP - 2}{2} \end{cases}$$

There is a third element of charges. For two funds (Inbursa and Atlantico) charges apply to the real rate of return. Thus, we need to modify the formula to incorporate that element.

Therefore, if we include charges on the real interest rate, the formula becomes

$$S_k = \begin{cases} \left(6.5\% * BW * 2 * \left(1 - \frac{CW}{6.5\%} \right) + GC \right) * \left((1 + i_1^{(12)}) * \left(1 - \frac{CB}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY \right) & k = 1 \\ S_{k-1} * \left((1 + i_1^{(12)}) * \left(1 - \frac{CB}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY \right) & k = 2i \quad i = 1, 2, \dots, \frac{CP}{2} \\ \left(S_{k-1} + \left(6.5\% * BW * 2 * \left(1 - \frac{CW}{6.5\%} \right) + GC \right) \right) * \left((1 + i_1^{(12)}) * \left(1 - \frac{CB}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY \right) & k = 2i + 1 \quad i = 1, 2, \dots, \frac{CP - 2}{2} \end{cases}$$

where $\pi^{(12)}$ is the monthly interest rate, and CY is the charge on the real interest rate and $i_R^{(12)}$ is the real interest rate

$$i_R^{(12)} = \frac{(i^{(12)} - \pi^{(12)})}{1 + \pi^{(12)}}$$

One assumption made here is that the charges remain fixed for the total life of the system. Charges for each company depends on the number of years a person has been in the AFORE. For example, AFORE Banamex charges 1.70% of wages up to year 4. However, for a person who stays with it for the fifth year gets a reduction in charges. Thus, year 5 charge becomes 1.68% of wages, year 6 charge becomes 1.66% of wages and so on. This process continues until year 39 with the AFORE with a reduction of 0.02% of wages for every additional year. Hence, our formula needs to take such a reduction into account.

$$S_k = \begin{cases} \left(\left(6.5\% * BW * 2 * \left(1 - \frac{CW * (1 - f_k)}{6.5\%} \right) + GC \right) * \right. \\ \left. \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) \right) & k = 1 \\ S_{k-1} * \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) & \\ k = 2i \quad i = 1, 2, \dots, \frac{CP}{2} & \\ \left(S_{k-1} + \left(6.5\% * BW * 2 * \left(1 - \frac{CW * (1 - f_k)}{6.5\%} \right) + GC \right) \right) * & \\ \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) & \\ k = 2i + 1 \quad i = 1, 2, \dots, \frac{CP - 2}{2} & \end{cases}$$

However, f_k is not the same for all funds. For example, AFORE Bancomer offers a rising discount rate starting with 0.01% of wages up to 0.05% of wages.

A MORE REALISTIC REPRESENTATION

There is still one realistic element missing in our formula: growth in wages. In Chile, the average wage rate has grown at a rate of 6% per year over the last twenty years. But, the rise in average wage rate is not important here as it represents the average across many individuals at a given point of time. For individuals, the more meaningful number is the growth of wage rate longitudinally. Therefore, we need to modify our formula thus:

$$S_k = \begin{cases} \left(\left(6.5\% * BW * 2 * \left(1 - \frac{CW * (1 - f_k)}{6.5\%} \right) + GC \right) * \right. \\ \left. \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) \right) & k = 1 \\ S_{k-1} * \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) \\ k = 2i \quad i = 1, 2, \dots, \frac{CP}{2} \\ \left(S_{k-1} + \left(6.5\% * BW * (1 + \Delta s^{(6)}) * 2 * \left(1 - \frac{CW * (1 - f_k)}{6.5\%} \right) + GC \right) \right) * \\ \left((1 + i_1^{(12)}) * \left(1 - \frac{CB * (1 - f_k)}{12} \right) - \left(\frac{i^{(12)} - \pi^{(12)}}{1 + \pi^{(12)}} \right) * CY * (1 - f_k) \right) \\ k = 2i + 1 \quad i = 1, 2, \dots, \frac{CP - 2}{2} \end{cases}$$

where $\Delta s^{(6)}$ is the bimonthly growth rate of wage rate of an individual worker over his or her lifetime. Here, we are assuming that the growth rate is constant. However, because of the recursive nature of the formula, it is easy to incorporate non-linear growth rate in wages. In some countries (Chile, South Korea), the average wage rates have risen by more than 6% in real terms per year. In others (Mexico), the average real wage rate has fallen over the past decade. However, here we should be looking at wage rate for each individual *longitudinally* and not the average wage for the population.

ASSUMPTIONS

Rate of return (Interest rate): We consider three regimes with real rates of return of 3%, 6% and 9%. Based on historical rates of return on portfolios that are heavily weighted to

government bonds, 3% represents a conservative figure, 6% an average figure and 9% an optimistic figure. Based on a history of 18 months, it is difficult to project rates of return on all funds. So, we assume them to be the same. That is, at a given point in time, we assume that all funds have the same (fixed) rate of return.

Wage rate: We consider wage rates in multiples of the minimum wage. This is customary in Mexico. Moreover, the government contribution is actually tied to the (real) minimum wage as of July 1, 1997 (see below). We consider (1) a person with minimum wage- obviously, this is of significance as a benchmark (as government contribution is based on this rate. (2) wage rate of 3.4 times the minimum wage - this is the average wage rate in Mexico. (3) 7.5 times the minimum wage (top 10% workers) and (4) 15 times minimum wage (contributions are capped at this rate). In addition, we also consider time paths with changing wage rate:

1. CONSTANT: The worker maintains the same wage, in real terms, during the working life.
2. LINEAR: The worker's wage increases 1.5% each year.
3. EXPONENTIAL: Under this assumption the wage increases according the following formula:

$$f(x) = \ln(x) \cdot a$$

Where:

$$a = \begin{cases} 4.5 & \text{if } x < 6 \\ 6 & \text{if } x > 6 \end{cases}$$

x = number of years working.

Cuota Social: The minimum wage actually is 34.45 pesos daily which means 1,033.50 pesos monthly and it's considered to remain the same during the whole analysis period. Thus, we take 5.5% of cuota social (indexed on July 1, 1997) as the amount of government contribution.

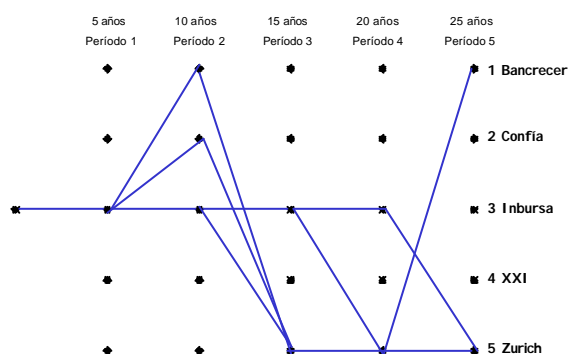
The model developed for this paper had to avoid the dimensionality problem mentioned earlier. To consider all the possible paths means that for each year the worker is in the system we have to calculate the final balance for each AFORE. For the first year, there are fourteen. Each one of these results become to be the initial balance for the second year and each one produce another fourteen different results as final balance for the second year so, we get fourteen times fourteen results in the second year. This procedure is recursively as time goes by. Once all the calculations are done we have to find the maximum of the final balances in the last year and reach its path. The problem here is that for a forty years of analysis the number of final balances is $7'000'376'965'910.7 \times 10^{35}$. For this analysis we have assumed that a worker change his or her AFORE once every five years. This last assumption reduces considerably the number of possible paths from $7'000.37 \times 10^{42}$ to $390'625$ which is easier to handle in terms of computational time.

RESULTS

The results shown in the next Table were obtained considering four different individuals of 41 years each with different initial wages (1, 3,4,7.5 and 15 minimum wages respectvely). We assume that all of them retire at 65 which means that they spend 25 years contributing to the system. All the salary behaviors were considered and the return rate was considered constant with three different options (3%, 6% and 9%) which gives us 36 different cases.

Trajectory Optimal

	Age					
	41	45	50	55	60	65
Bancrecer			■			■
Principal			■			
Inbursa	?		■	■	■	
XXI						
Zurich				■	■	■



From these results we can see that none of the 36 paths reach AFORE XXI on any period. It is clear that after the second period (year 10) this AFORE is not an option. The reason is that Bancrecer charges less than XXI by flow and the last one also charges by balance so it become more expensive after this point and a rational worker will never choose this AFORE. This reason allows us to simplify a little more our analysis because we take out XXI of the five AFOREs we were considering and the following results only consider the other four AFOREs. This simplification reduces the number of possibilities in a forty years period from 325'089 to 65'536.

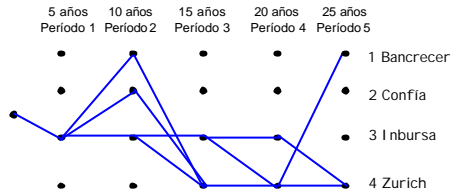
From now on all the results shown only consider this four AFOREs: Bancrecer, Principal, Inbursa and Zurich. We consider the same assumptions for the results and we simulate for 20, 25, 30, and 35 years.

25 Años Cotizados

Rendimiento Real	Salario Inicial	Función de Crecimiento Salarial	Trayectoria Óptima
3%	1	Exponencial	33334
6%	1	Exponencial	33444
9%	1	Exponencial	32444
3%	1	lineal	33444
6%	1	lineal	32441
9%	1	lineal	32441
3%	1	constante	33441
6%	1	constante	32441
9%	1	constante	31441
<hr/>			
3%	3.4	Exponencial	33334
6%	3.4	Exponencial	33444
9%	3.4	Exponencial	32444
3%	3.4	lineal	33344
6%	3.4	lineal	32444
9%	3.4	lineal	32441
3%	3.4	constante	33344
6%	3.4	constante	32441
9%	3.4	constante	32441
<hr/>			
3%	7.5	Exponencial	33334
6%	7.5	Exponencial	33444
9%	7.5	Exponencial	32444
3%	7.5	lineal	33344
6%	7.5	lineal	32444
9%	7.5	lineal	32441
3%	7.5	constante	33344
6%	7.5	constante	32444
9%	7.5	constante	32441
<hr/>			
3%	15	Exponencial	33344
6%	15	Exponencial	33444
9%	15	Exponencial	32441
3%	15	lineal	33344
6%	15	lineal	33444
9%	15	lineal	32441
3%	15	constante	33344
6%	15	constante	32444
9%	15	constante	32441

Trayectoria Óptima

	Edad					
	41	45	50	55	60	65
Bancrecer						
Confia						
Inbursa	?					
Zurich						

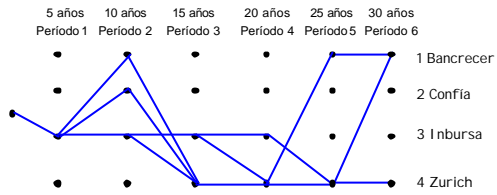


30 Años Cotizados

Rendimiento Real	Salario Inicial	Función de Crecimiento Salarial	Trayectoria Optima
3%	1	Exponencial	333344
6%	1	Exponencial	334441
9%	1	Exponencial	324441
3%	1	lineal	334441
6%	1	lineal	324411
9%	1	lineal	324411
3%	1	constante	334411
6%	1	constante	324411
9%	1	constante	314411
<hr/>			
3%	3.4	Exponencial	333344
6%	3.4	Exponencial	334441
9%	3.4	Exponencial	324441
3%	3.4	lineal	333441
6%	3.4	lineal	324441
9%	3.4	lineal	324411
3%	3.4	constante	333441
6%	3.4	constante	324411
9%	3.4	constante	324411
<hr/>			
3%	7.5	Exponencial	333344
6%	7.5	Exponencial	334441
9%	7.5	Exponencial	324441
3%	7.5	lineal	333441
6%	7.5	lineal	324441
9%	7.5	lineal	324411
3%	7.5	constante	333441
6%	7.5	constante	324441
9%	7.5	constante	324411
<hr/>			
3%	15	Exponencial	333441
6%	15	Exponencial	334441
9%	15	Exponencial	324411
3%	15	lineal	333441
6%	15	lineal	334441
9%	15	lineal	324411
3%	15	constante	333441
6%	15	constante	324441
9%	15	constante	324411

Trayectoria Optima

	Edad						
	36	40	45	50	55	60	65
Bancrecer							
Confía							
Inbursa							
Zurich							

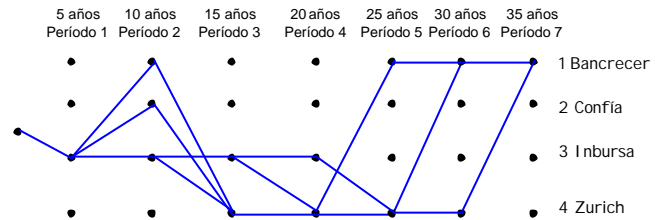


35 Años Cotizados

Rendimiento Real	Salario Inicial	Función de Crecimiento Salarial	Trayectoria Óptima
3%	1	Exponencial	3333441
6%	1	Exponencial	3344411
9%	1	Exponencial	3244411
3%	1	lineal	3344411
6%	1	lineal	3244111
9%	1	lineal	3244111
3%	1	constante	3344111
6%	1	constante	3244111
9%	1	constante	3144111
<hr/>			
3%	3.4	Exponencial	3333441
6%	3.4	Exponencial	3344411
9%	3.4	Exponencial	3244411
3%	3.4	lineal	3334411
6%	3.4	lineal	3244411
9%	3.4	lineal	3244111
3%	3.4	constante	3334411
6%	3.4	constante	3244111
9%	3.4	constante	3244111
<hr/>			
3%	7.5	Exponencial	3333441
6%	7.5	Exponencial	3344411
9%	7.5	Exponencial	3244411
3%	7.5	lineal	3334411
6%	7.5	lineal	3244411
9%	7.5	lineal	3244111
3%	7.5	constante	3334411
6%	7.5	constante	3244411
9%	7.5	constante	3244111
<hr/>			
3%	15	Exponencial	3334411
6%	15	Exponencial	3344411
9%	15	Exponencial	3244111
3%	15	lineal	3334411
6%	15	lineal	3344411
9%	15	lineal	3244111
3%	15	constante	3334411
6%	15	constante	3244411
9%	15	constante	3244111

Trayectoria Óptima

		Edad							
		31	35	40	45	50	55	60	65
Bancrecer									
Confía									
Inbursa	?								
Zurich									



CONCLUSIONS

From these calculations, we can conclude that usually there are three stages in a worker's life. In the beginning, the balance in his individual account is small. In this case, the best option is always Inbursa because it charges a percentage of the real return and this commission is going to be less than the others in the market.

At the end of the period it is also clear that the best option is an AFORE that charges only by flow. This is because when the balance in the individual account becomes significant, charges by flow makes a big difference.

Finally there is an intermediate period where is not that clear what the best option is. Here, the optimum choice depends on the personal characteristics of the worker. However, it is clear that there are a period in which the best option is to be charged only by balance. This is the case of AFORE Zurich because it has a discount scheme and after ten years it commission is going to be the lower in the market.

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