Leonid Hurwicz and the Term "Bayesian" as an Adjective

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Abstract
Leonid Hurwicz was the first person to have used the term Bayesian as an adjective. The usage has been documented since 1950.

Key words: Bayesian Inference; Neo-Bayesian revival; Axiomatization; Rational behavior; Economics; Statistics; Nobel Prize.

Introduction

Fienberg (2006) justly wrote the first paper of the first number of the electronic journal Bayesian Analysis. His purported purpose was to elaborate on the term "Bayesian" as an adjective. In the process of this elaboration, Professor Fienberg produced a fine analysis of the history of what we know today as Bayesian Statistics. His bibliography is extensive with 184 items. Some of them (like his quotations from Jack Good and John Pratt) are in the form of personal correspondence.

He takes us on a marvelous journey of two hundred years of history of Bayes and Bayesian thinking. He divides the history into three distinct phases. First came Bayes and Laplace with a reminder of Stigler's Law of Eponymy ("no scientific discovery is named after its original discoverer"). Then, as a precursor to the modern development, he discussed contributions during the first five decades of the Twentieth Century. Finally, he brought in the Neo-Bayesian Revival of the 1950s.

Fienberg quotes from Lindley (2000), "When I began studying statistics in 1943 the term 'Bayesian' hardly existed; 'Bayes' yes, we had his theorem, but not the adjective." Fienberg also notes that Good (1950) "writing on the weighing of evidence using Bayes' Theorem, in the third paragraph of the preface used the phrase 'subjective probability judgments,' but nowhere in the book did he use the adjective 'Bayesian'."

Fienberg notes that the usage of the term "Bayesian" was published during 1950-1951: One by Ronald Fisher in 1950 (in a pejorative way) and by Jimmy Savage in 1951 where he used the phrase "unBayesian" (Savage, 1951, p. 58). Neither usage would count as we use the term today as an adjective. Fienberg then writes "[a] search of JSTOR reveals no earlier usage in any of the main American and British statistical journals." He then goes on to suggest that it was Jimmy Savage that brought the Bayesian adjective to the fore.

In what follows, we will discuss the role that economists have played during the critical period of the development of the Neo-Bayesian Revival. In particular, we point out the contribution of the 2007 Nobel Prize winner in Economics, Professor Leonid L Hurwicz.

Jimmy Savage and Economics

Fienberg justifiably examines Savage's book "The Foundation of Statistics" as a milestone for the Neo-Bayesian Revival. It is striking that the first five chapters of the book could very well have been called "The Foundation of Economic Theory Under Risk and Uncertainty." It begins with the (Expected) Utility Theory – which has become the cornerstone of modern micro and macro economic theories of today.

Savage's book did more than provide the foundation of modern economic theory. His book (unwittingly) propagated an expansion. For example, in 1952 in a well known encounter, at the Econometric Society's meeting in Paris, Maurice Allais presented Savage (and other participants) with the following hypothetical choices.

Situation 1: Choose between

Gamble 1: \$500,000 with probability 1; and
Gamble 2: \$2,500,000 with probability 0.1,
\$500,00 with probability 0.89,
status quo with probability 0.01.

Situation 2: Choose between

Gamble 3: \$500,000 with probability 0.11,
status quo with probability 0. 89; and
Gamble 4: \$2,500,000 with probability 0.1,
status quo with probability 0.90.

Savage chose gambles 1 and 4 respectively. Allais noted that it contradicted expected utility theory – the very foundation of a rational decision maker of The Foundations of Statistics! As we all know, in the book, Savage goes on to describe why such a choice was not rational (even though he *himself* made the choice initially).

A small exchange like this could have just been a curiosum – a footnote to the history of economics. But it did not turn out that way. Allais went on to develop the theory of Non-Expected Utility for which he and later Daniel Kahneman went on to win Nobel Prizes in Economics (Amos Tversky – the

close collaborator of Kahneman would also have won the prize had he not died before the prize was awarded).

Thus, we contend that Jimmy Savage strongly influenced the critical development of economic theory as a whole. To prove our point, we did some bean-counting. There were slightly less than one thousand references in JSTOR of the book by Savage (ending in 2006). Of them around 30 percent were in economics journals. For sure, some of them occurred in journals that were right in the intersection of these two disciplines: Economics and Statistics. A classic example is Chernoff (1954). He refers to Savage's "Notes on the Foundations of Statistics" with the comment that it would be published in a book form. In turn, Savage's research was also influenced by economics. To wit, around a quarter of his references are directly from economics literature.

In contrast to mainstream economics, we all know that Savage's book was not very well received by many classical statistical theorists. For example, Chung (1955) reviewed the book with the following comment. "A book like this is necessarily part philosophy, and one who is not philosophically bent, as Mr. Savage clearly is, is often hard put to tell between what is critical thinking and what is quibbling about words. To such a person a good part of the discussion of the foundations of probability is typified by the following two examples. 1. Re probability: when a coin is tossed there is besides head and tail the possibility of the coin's standing on its edge or disappearing into a crevice. (For a variation on this theme see p. 15 on whether a rotten egg spoils an omelet.) 2. Re utility: some people gamble for a monetary loss in order to kill time or to cultivate good relations. (For a variation see p. 101 on the show-off flier.) I do not know how to draw a line between such bull-session stunts and more serious argumentation…"

To contrast the impact of Savage's book on Statistics, we counted the number of citations it received in Statistics literature. The total count in JSTOR is 483 (until the end of 2006). Of them, around 24 percent occurred during the 1980s – the time during which Bayesian concept got a big boost from computational breakthroughs. It must be recognized that JSTOR citations in economics versus statistics can be tricky. There are twice as many economics journals in JSTOR.

The Cowles Commission in Chicago

Nobel Prizes in Economics have been awarded since 1969. And the list of names participating in the Cowles Commission who have won the Nobel Prize is long. The list includes (with the year of the award in parenthesis): Ragner Frisch (1969), Tjalling Koopmans (1975), Kenneth Arrow (1972), Herbert Simon (1978), Gerard Debreu (1983), Maurice Allias (1988), Franco Modigliani (1985), Harry Markowitz (1990), Trygve Haavelmo (1989), James Tobin (1981), Edmund Phelps (2006), Joseph Stiglitz (2001), Lawrence Klein (1980) and Leonid Hurwicz (2007). Thus, about a quarter of all Nobel Prize winners in Economics were associates of the Cowles Commission. It is also noteworthy that Savage's book includes references to three members of this Cowles-Nobel list above (Arrow, Allais and Markowitz).

The Cowles Commission emerged as the second most influential institution in economics (after the National Bureau of Economic Research) both at the policy level and more importantly, for its contribution to economic theory. In the late 1940s and early 1950s, it was a hotbed of research in both economics and statistics. Thus, it is not surprising to find that researchers there were intensely working on Bayesian issues. In particular, Leonid Hurwicz was working on Bayesian formulations of decision making under risk and uncertainty. More on Bayesian work at the Cowles Commission can be found in Fienberg and Zellner (1975).

The Cowles Commission operated at the University of Chicago between 1939 and 1955, before moving to its permanent home at Yale University. Hildreth (1981) provides a history of the Commission during this period.

Leonid Hurwicz and the Bayesian adjective

Hurwicz was a remarkable man. He was born in Moscow in 1917, grew up in Poland, survived the Holocaust just by a hair, took the last boat to leave Europe to arrive in New Jersey with no money and a degree in Law from the University of Warsaw. That was his only degree he ever earned (apart from honorary degrees from a number of universities). Yet he went on to become one of the foremost economic theorists of the Twentieth Century and the oldest recipient of a Nobel Prize in any subject (at the age of 90). He also

became the only person to have received a Nobel Prize without ever formally studying economics (Sinha, 2008).

Before Savage joined the University of Chicago's newly founded Statistics Department in 1949, Hurwicz was already there. He became a Research Associate at the Cowles Commission in 1942 (http://cowles.econ.yale.edu/P/reports/1942.htm). During the war, Hurwicz was moonlighting: teaching electronics to the U.S. Army Signal Corps at the Illinois Institute of Technology. At the University of Chicago, he was a member of the faculty of the Institute of Meteorology and taught statistics in the Department of Economics. He worked under Jacob Marschak and Tjalling Koopmans at the Cowles Commission for Research in Economics and Statistics at the University of Chicago.

His work overlapped with what Savage was doing. This fact is evident from the Cowles Commission Annual Report of 1950-51: "…[M]any statisticians feel that, in their own practice, they have to choose a 'decision function' (i.e., they have to design a sample or an experiment and derive in advance a formula relating action to observation) without any advance knowledge as to the relative probabilities of alternative states of nature. The same is true of many practical situations. In fact, only in exceptional cases (such as life insurance, games of chance, and scientific predictions based on much past experience) does the decision-maker have good information on the relevant probabilities. In the general case, such information is not available; hence moral expectation cannot be computed. Additional criteria become necessary. Thus a pessimist will assume the worst possible state of nature to be true and hence will maximize the lowest possible moral expectation; while, as pointed out by Franco Modigliani, the optimist will maximize the maximum moral expectation. Leonid Hurwicz formulated a certain compromise between the two attitudes. In general, the compromise may be slanted toward optimism or pessimism, the extent of the slant being part of a person's 'tastes.' Another criterion was suggested by L.J. Savage and, independently, by Jurg Niehans of Zurich: for any given state of nature define as 'loss' (or 'regret') the difference between the highest moral expectation that could be obtained if that state were known and the moral expectation obtained from a given action; then choose the action for which the highest loss is lower than for any other action." (http://cowles.econ.yale.edu/P/reports/1950-51a.htm)

In the paper referred to in the previous paragraph, Hurwicz (1951a) introduced his famous "alpha" that mitigated between minimax and maximax rules of decision. Milnor (1951) in his now famous RAND Research Memorandum expanded upon this rule. On page 2 of the same paper, Hurwicz notes: "The solution has been called 'Bayesian' (or 'Bayes Optimal') with regard to $H^{(0)}(b)$." Thus, already in February of 1951, we have documented proof that Hurwicz was using the term Bayesian as an adjective. In a subsequent Discussion Paper, Hurwicz (1951b) mentions "the Bayesian case" once more. Thus, not only was he using the phrase Bayesian as an adjective, he was also anticipating the difference between Bayesian and non-Bayesian cases in the ambit of decision making under uncertainty. In footnote 1 of the same paper, he notes, "The more usual procedure is first to form a 'risk function' $\rho(I_F, \Psi)$ with ρ depending on the statistician's preferences when $\vartheta \ni *$ is of the Bayesian type".

Is that the earliest reference to a clearly documented Bayesian we find in the Neo-Bayesian Revival movement? The answer is negative. In a paper dated December 25, 1950, Hurwicz (1950a) develops a technique of estimation using Bayes Theorem. In discussing the method, he remarks, "The foregoing techniques … can be applied to justify (from the Bayesian point of view) the maximum likelihood method of estimation of the mean μ of a normal distribution with a known variance σ^2".

At around the same time, Hurwicz (1950b) noted "At the opposite extreme there exists the 'Bayesian' formulation, where it is assumed that a probability measure ξ on ϑ (an 'a priori distribution') is known to the statistician."

The main contribution, for which Hurwicz shared his Nobel Prize with two others (Roger Myerson and Eric Maskin), was pioneering work on Mechanism Design. He also received the National Medal of Science in 1990 in Behavorial and Social Science "for his pioneering work on the theory of modern decentralized allocation mechanisms". He became the only economist to receive that honor *before* winning the Nobel Prize. This shows the diversity of Hurwicz's research.

In more than one occasion, researchers have discovered results only to find that Hurwicz was there first. He was a true scientific "Kilroy". For example, in an interview, Jack Good once noted that after he introduced the notion of

hierarchical Bayesian analysis: “the econometrician, L. Hurwicz, turned out to have published an abstract a few months before my 1951 paper, suggesting the minimax example…” (Banks, 1996).

Not only did Hurwicz use the term “Bayesian” as an adjective in his research papers in the early part of 1950s, he was already using these notions for the course he was teaching in the statistics department at the University of Minnesota. The following paragraph reproduces the first question in the PhD prelim examination in December 1953 written by Hurwicz.

> *A has two coins (c_1, c_2) of identical appearance but different weight and weight distribution. B his permitted to observe one of the coins and is then required to guess whether it was c_1 or c_2. He knows that the probability of heads is 1/3 for c_1 and 3/4 for c_2.*
>
> *(a) List all the possible non-randomized decision functions;*
> *(b) Indicate inadmissibility if found;*
> *(c) Find the maximum likelihood solution;*
> *(d) Find a Bayesian solution;*
> *(e) Find a minimax solution.*
>
> *(In the later two cases, make such additional assumptions as necessary.) Show that (c) is a special case of Bayesian solution.*

This example shows that Hurwicz propagated the notion of Bayesian Statistics to the next generation of students. Some of these students went on to become outstanding econometricians of their own rights. Indeed, one such shining example was Daniel McFadden who went on to win a Nobel Prize in Economics for his work on discrete choice econometrics.

Final Remarks

We provided evidence that Leonid Hurwicz might be the first person to have used the term Bayesian as an adjective. We have shown that during the Neo-Bayesian Revival, a strong interaction took place among Economists and Statisticians: the ideas of an axiomatic foundation for the rational behavior of an economic agent on one hand, as well as for the coherent production of statistical inferences, on the other. They were essentially variations of the same theme. In the case of Economics, the resulting criteria of maximizing

the expected utility came to occupy the central stage of modern (neoclassical) economic theory. On statistical theory, the effect was somehow different. The Neo-Bayesian Revival led to a new paradigm: the axiomatic development of Bayesian Theory of Statistics.

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