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CAPITAL ACCUMULATION AND LONGEVITY

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Increased survival probability and expected lifespan is shown to increase capital intensity and welfare in an overlapping generations model with uncertain lifetimes in the presence of annuities market and production.

1. Introduction

Economic development is usually accompanied by marked growth in life expectancy of the population and lower mortality rates. The effects of economic development on the population demographics have been studied extensively in the economics literature. Recently, we are witnessing a growing awareness among economists regarding the effects of demographic changes on the growth and development of the economic models [see, for example, Clark et al. (1978)]. In most countries, a rise in life expectancy has been caused by the lowering of mortality rate of babies and children. But, for higher age groups, this increase has been extremely small. For example, in the U.S. between 1900 and 1980, life expectancy at birth grew by 26 years whereas life expectancy at age 75 grew by only three years [Fries (1980)]. It has been noted in the literature that such increases in life expectancies lead to greater (precautionary) saving [Abel (1985)] on the part of the young workers, but to date, full implications of such increases in saving have not been traced out in a general equilibrium model. The only rigorous attempt to date was made by Kotlikoff (1979). He studied the effects of expanded lifespan on the capital accumulation in a neoclassical growth model. In his model, 'expanded lifespan' is interpreted as an increase in the total lenght of (certain) lifetime together with an increase in the number of working years. With specific utility and production functions, he shows that an 'expanded lifespan' leads to an increase in capital accumulation. There are several drawbacks with this approach. First, the working years of the population in the developed countries seem to be decreasing (at least not increasing). Second, life expectancy of the 'very old' (say, individuals above the age of 90) has increased marginally. Therefore, expanded lifespan should not be treated as an increase from two to three periods of certain lifetime. A more plausible model should be a model with uncertain lifetimes, where the survival probability for the lower age group is increasing but the probability of survival for the oldest age group remains unchanged. This is the approach followed in this paper.

We may also note that the uncertain lifetimes model used in this paper has been very useful in explaining the aggregate capital accumulation in the economy [Davies (1981)]. A model of uncertain

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lifetimes, together with a bequest motive has been used in an ingenious paper by Skinner (1985) to explain a *fall* in the saving rate in the U.S. in the seventies in the face of rising life expectancies.

The effects of increased longevity can be analyzed as follows. First, increased longevity will affect the behavior of saving in the economy. This will in turn alter the capital accumulation in the long run through changes in the wage rate and real interest rate. Thus, the welfare of the individuals in the steady state will be affected.

To get a clear answer, we make a number of simplifying assumptions regarding the economy. These assumptions are spelled out in section 2. Section 3 discusses the impact of a change in the distribution of survival probabilities in the population on the steady state capital accumulation and the welfare in the economy. The final section indicates the directions for future research.

2. The model

We carry out the analysis in the framework of Diamond (1965). However, we extend the model to incorporate uncertain lifetimes. The economy is specified as follows:

(*a*) *The population.* At each discrete date t ($t \geq 1$), a new generation appears; we call them generation t. Each member of generation t is alive at t, but she faces uncertainty at the beginning of period $t+1$: she dies with probability $1-p$ ($0<p<1$) at the beginning of period $t+1$. However, if an individual of generation t does not die at the beginning of period $t+1$, she does not die until the end of period $t+1$. At the end of period $t+1$, all surviving members of generation t face certain death. Before an individual of generation t faces death at the beginning of period $t+1$ she gives birth to another individual. Thus, if there are N identical individuals born at time t, then there are $N+Np$ individuals alive at every date t.

(*b*) *Preference.* Each individual of generation t maximizes a von Neumann–Morgenstern utility function,

$$W(c_t, c_{t+1}, p) = u(c_t) + pv(c_{t+1}), \tag{1}$$

where c_t, c_{t+1} are the consumption bundles of the individual of generation t in period t and $t+1$ respectively.

(*c*) *Technology/Annuity.* There is an available technology which converts t-good into $(t+1)$-good as follows: let y be the output per unit of labor and k be the capital per unit of labor. Then $y=f(k)$ with $f'(x) \geqq 0$ and $f''(x)<0$ for all $x \geqq 0$. We assume that each member of generation t works during t and buys a contract from the firm (with the saving), which acts as a pension fund. Therefore, the individuals of generation t get a return on their investment only if they are alive to enjoy the benefits during $t+1$.

(*d*) *Endowments.* Each member of generation t is endowed with one unit of labor at t. If an individual of generation t is alive at $t+1$, she cannot work during $t+1$.

(*e*) *Market structure.* We assume that the markets for labor and capital/annuity are perfectly competitive. Therefore, the competitive wage w is such that

$$w(k) = f(k) - kf'(k) \tag{2}$$

and the gross rate of return to capital R is such that

$$R(k) = 1 + f'(k). \tag{3}$$

Given that the competitive contract specifies the return only for the surviving members of every generation, an individual of generation t gets a gross rate of return R/p in period $t+1$ per unit saving in period t. Now we have completely described the economy.

In a steady state, a newborn individual solves the following problem:

$$\max_{\{0 \leq k \leq w\}} u(w-k) + pv(Rk/p). \tag{4}$$

She treats w and R parametrically. For the existence of a unique solution k^* to the problem (P), we postulate the following assumption:

A.1. $u'(x)$, $v'(x) > 0$ and $u''(x)$, $v''(x) < 0$ for every $x > 0$;

$$\lim_{x \to 0} u'(x) = \lim_{x \to 0} v'(x) = \infty \quad \text{and} \quad \lim_{x \to \infty} u'(x) = \lim_{x \to \infty} v'(x) = 0.$$

3. The effects of increased longevity

With the model described above, we can precisely pose the questions of section 1: (a) how will a change in p affect k^*? (b) how will a change in p affect W?

Let us first note that the answers are not obvious. A change in p will alter the saving behavior of an individual in the steady state which will in turn alter the capital accumulation pattern in the economy. Therefore, the wage rate and the real interest rate will alter too.

The first-order condition of the problem (4) is given by

$$u'(w^* - k^*) = R^* v'(R^* k/p), \quad \text{where} \tag{5}$$

$$w^* = f(k^*) - k^* f'(k^*) \quad \text{and} \quad R^* = 1 + f'(k^*).$$

We differentiate (5) totally and get the following expression:

$$u''[\mathrm{d}w - \mathrm{d}k^*] - \mathrm{d}R[v' + Rk^* v''/p] - R^2 v'' \,\mathrm{d}k^*/p + R^2 k^* v'' \,\mathrm{d}p/p^2 = 0. \tag{6}$$

Differentiating the eqs. (2) and (3), and inserting the result in (6), we get

$$-u''[1 - k^* f''] - k^* f'' v'[1 - r_v] = -(R^2 k^* v''/p) \,\mathrm{d}p/\mathrm{d}k^*, \quad \text{where} \tag{7}$$

$$r_v = -xv''(x)/v'(x), \quad \text{with } x = Rk^*/p.$$

Clearly, under A.1, the sign of the right-hand side of (7) is positive. It is possible to determine the sign of the left-hand side of (7).

Proposition 1. If r_v is less than unity, then a rise in the probability of survival leads to higher capital accumulation.

Table 1
Value of capital intensity (k). [a]

	$p=0.5$	$p=0.6$	$p=0.7$
$\beta=2$	0.05441	0.07208	0.08948
$\beta=3$	0.03643	0.05260	0.06920

[a] The value of α for the above calculations is 0.3.

Proof. Notice that in (7), all terms in [] are positive with the assumption of the proposition. Therefore, the proposition follows. Q.E.D.

How large are these effects of increased longevity on the capital intensity in the economy? To get an answer, I need to assume functional forms of u, v and f. For illustrative purposes, I assumed $u(x)=v(x)=x^{1-\beta}/(1-\beta)$ and $f(k)=k^{\alpha}$.

Table 1 gives typical simulation results obtained.

An interesting feature of the simulation is the extent of increase in capital intensity due to an increase in probability of survival. A 15% rise in the probability of survival leads to an increase of 20 to 30% rise in the capital intensity. Therefore, the effects of survival probability on the capital accumulation is substantial.

I shall now address the second question posed in the beginning of this section dealing with the welfare of the individual given an increase in the probability of survival. More specifically, given that a change in p is positively associated with a change in k^*, we can establish the following:

Proposition 2. Suppose $\mathrm{d}k^*/\mathrm{d}p>0$ *and* $v(0)>0$. *Then, a rise in the survival probability will lead to a rise in the welfare of the individual in the steady state, i.e.,* $\mathrm{d}W/\mathrm{d}p>0$.

Proof.

$$\begin{aligned}\mathrm{d}W/\mathrm{d}p &= [\partial w^*/\partial p-\mathrm{d}k^*/\mathrm{d}p].u'+v\\ &\quad+\left[-R^*k^*/p^2+(k^*/p)\ \partial R^*/\partial p+(R^*/p)\ \mathrm{d}k^*/\mathrm{d}p\right].v'\\ &=v(x)-xv'(x)+k^*f'[v'-u']\ \mathrm{d}k^*/\mathrm{d}p, \quad \text{where } x=R^*k^*/p.\end{aligned}$$

Now, from the first-order condition of the optimizing problem (P), $v'-u'=-f'v'$. Therefore, by hypothesis of the proposition, the second term is positive since $f''<0$. By mean value theorem, $[v(x)-v(0)]/[x-0]=v'(y)$ for some $0<y<x$. Therefore, $v(x)=v(0)+xv'(y)$ and $v(x)-xv'(x)=v(0)+x[v'(y)-v'(x)]$. But, $0<y<x$ and $v''<0$ by A.1. Thus, $v'(y)-v'(x)>0$ provided the first term on the right-hand side of $\mathrm{d}W/\mathrm{d}p$ is positive. Q.E.D.

In the discussion above, we have used the terms 'a change in longevity' and 'a change in the distribution of survival probability' interchangeably. This was possible because in the two-period model expected length of life and survival probability have a linear relationship [because expected length of life is, by definition, $1.(1-p)+2.p=1+p$].

We have restricted our discussion to the case where the gross rate of return on annuities is R/p, i.e., the annuities market is actuarially fair. Do the results above change if the annuities market is *not* actuarially fair? It can be shown by a little more tedious algebra that these results do extend to the case of annuities markets which are not actuarially fair.

4. Conclusion

As it was noted in the introduction there seems to be very little in the literature which discusses the economic effects of changes in the lifespan. Kotlikoff (1979) seems to be the only other study in the literature directly dealing with the economic effects of longevity. He studies the effect of an expanded lifespan on the capital accumulation in an economy. He also shows that for specific functional forms of utility and production, and for range of parameteric specifications, the capital intensity increases with increased lifespan. However, in his model *there is no uncertainty in the life of the agents* and consequently no role of annuity in the model. The phrase 'expanded lifespan' in his model is interpreted as an increase in the total length of (certain) lifetime together with an increase in the number of working years. Moreover, unlike the present paper, he does not address the issue of changes in welfare due to the change in the lifespan of the agents.

Future research in this area should look at the following question. Is saving positively related to increased life expectancy over time (in a given country) or across countries at a given time? Several things have to be controlled to do such an empirical study: (a) taxes on saving, (b) price rise over time, (c) distortion in the saving figure due to distortions in the market for foreign exchange etc.

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