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Some Applications of a Model of Term Life Insurance Contracts

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Abstract

In this note, I have shown how economically meaningful propositions can be derived regarding the effects of human wealth, nonhuman wealth and the probability of dying on the optimal term life insurance coverage of an individual with state dependent preferences in the framework of Babbel & Economides (1985).

In an interesting paper in this journal, Babbel & Economides (1985) have investigated into the design of life insurance contracts. They derive a number of comparative statics results to show how optimal term life insurance coverage is affected by: (i) a loading factor; (ii) changes in human wealth; (iii) changes in nonhuman wealth. They also show that an allocation resulting from a contract with a "policy fee" is Pareto-superior to any allocation produced by a contract without a policy fee.

In this paper, I shall extend the comparative statics analysis of Babbel and Economides. These extensions will be achieved with additional restrictions on the utility function and on the market for term life insurance contracts.

Babbel & Economides derive the optimal contract for the insurance buyer by maximizing the state dependent utility (as a function of premium P and coverage I):

$$U(I, P) = (1-\pi)V(W+H-P)+\pi B(W+I-P) \tag{1}$$

subject to a linearity condition of the contracts purchased: $P=I/m$, where, V is the "living" utility function and B is the bequest utility function; π is the probability of death; W is the nonhuman wealth; H is the human wealth; m is the loading factor.

The necessary condition for maximizing (1) subject to the linearity of contracts is given by

$$[\pi/(1-\pi)](m-1)B'(W+I-P) = V'(W+H-P). \tag{2}$$

Note that (2) is the same condition as Babbel & Economides's equation (4). The crucial aspect of equation (2) is that B' is evaluated at $W+I-P$ and

V' is evaluated at $W+H-P$. Therefore, unless $H=I$, *B' and V' are not evaluated at the same point under an optimal contract.* Babbel & Economides show that the effects of human and nonhuman wealth on the optimal purchase of term life insurance depend on the relationship

$$A_V(x) = -V''(x)/V'(x) < A_B(y) = -B''(y)/B'(y)$$

where, $x=W+H-P$ and $y=W+I-P$, i.e., to find the direction of change of sign of dI^*/dH and dI^*/dW, Babbel & Economides compare two risk aversion functions A_V and A_B at two different points (because x is not equal to y in general). Under some additional assumptions, I show that the above condition can be derived from more economically meaningful conditions. First, I state the two additional assumptions needed for the results in this paper.

(*A1*): Bequest utility function B is a linear transformation of the "living" utility function V: $B(x)=kV(x)+c$, for some $k>0$ for all $x\geq 0$.

(*A2*): V displays *strictly decreasing* absolute risk aversion: $A_V(x)= V''(x)/V'(x)$ decreases monotonically as x increases.

(*A1*) has been used frequently in the economics literature (for example, see Blinder (1974, p. 37)) and it has also been used in the insurance literature (see, Campbell, 1980, for example). The parameter k is interpreted as the strength of bequest motive (Fischer, 1973). If $k<1$ ($k>1$), the individual derives less (more) marginal utility from the inheritor's consumption than his own consumption and we say that the strength of bequest motive is *low* (*high*). (*A2*) has been proposed by Arrow (1965). It has also been used extensively in the literature (for example, Sandmo, 1970).

In the rest of this paper, I shall derive conditions for which I can compare the magnitudes of coverage I^* and human wealth H (i.e., whether H is greater or less than I^*). Utilizing this relationship, I shall derive conditions which will allow me to determine the sign of dI^*/dH and dI^*/dW in terms of the "strength" of bequest motive. I shall also show that under the condition of risk aversion, the sign of $dI^*/d\pi$ can also be determined.

Let $T=[\pi(m-1)/(1-\pi)]k$.

First I prove a simple lemma which deals with the insurance coverage vis-a-vis human wealth in an obvious way. The result of this lemma will be used in proving the results.

Lemma. *Let a risk averse individual's state dependent utility function be given by (1). Under the assumption (A1), the optimal purchase of insurance coverage I^* has the following relationship with the human wealth H:*

$H>I^*$ *if and only if* $T<1$.

Proof. The condition (2) for maximizing (1) given (*A1*) reduces to

$$V'(W+H-P) = TV'(W+I-P). \qquad (3)$$

It follows from (3) that $T<1$

if and only if $V'(W+H-P)<V'(W+I^*-P)$
if and only if $H>I^*$ (given that $V''<0$). Q.E.D.

Note that the condition $T<1$ can be satisfied in a number of different ways. A *sufficient* (but not necessary) set of conditions are

(i) $k<1$, i.e., the strength of bequest motive is low;
(ii) $P\geqslant\pi I$, i.e., the market for insurance is actuarially fair or less than fair.

The lemma above tells us that the optimal term life insurance coverage will be strictly less than the human wealth if the strength of bequest motive is low and the term life insurance market is at best actuarially fair.

Proposition 1. *Let (A1) and (A2) hold for a risk averse individual. Then,* $T<1$ *if and only if* $dI^*/dH<1$.

Proof. From the footnote 10 of Babbel & Economides, $dI^*/dH<1$ if and only if $A_V(x)<A_B(y)$ (the definitions of A_V, A_B, x and y are given above). By the lemma above, $T<1$ if and only if $I^*<H$ if and only if $x>y$. By (A2), $x>y$ if and only if $A_V(x)<A_V(y)$. The proof is complete by noting that by (*A1*), $A_V(y)\equiv A_B(y)$. Q.E.D.

Proposition 2. *Let (A1) and (A2) hold for a risk averse individual. Then* $T<1$ *if and only if* $dI^*/dW>0$.

Proof. In footnote 11 Babbel & Economides have shown that $dI^*/dW>0$ if and only if $A_V(x)<A_B(y)$. The rest of the arguments follow from the proof of proposition 1. Q.E.D.

Proposition 1 shows that if (i) an individual's strength of bequest motive is low and if (ii) the market for term life insurance is at best actuarially fair, then the rise in the optimal insurance coverage is not as fast as the rise in the human wealth. Proposition 2 shows that under the same conditions (i) and (ii), a rise in the non-human wealth leads to an increase in the optimal insurance coverage. These propositions show the strengths and weaknesses of the condition (5) of Babbel & Economides more vividly. In particular, if the bequest motive is strong and the term life insurance is at least actuarially fair, condition (5) of Babbel & Economides will be violated.

Finally, I shall show that it is very easy to derive the effect of variation in the survival probability on the optimal purchase of term life insurance coverage in the Babbel & Economides model. To fix the idea, suppose there are two individuals with identical felicity functions V and B. Suppose the first individual has a (slightly) higher probability of dying than the second. Can we conclude that the first individual will buy more coverage than the second? The answer is affirmative if V and B display risk aversion.

Proposition 3. *Let V'', $B''<0$. Then $dI^*/d\pi>0$.*

Proof. Let $f(I, \pi)=(1-\pi)V'(W+H-P)+(1-m)\pi B'(W+I-P)$.
Then by (2), $f(I^*, \pi)=0$. By Implicit Function Theorem,

$$dI^*/d\pi=-(\partial f/\partial\pi)/(\partial f/\partial I^*)=[V'+(m-1)B']/[(1-\pi)(-1/m)V''+(1-m)(1-\pi)B'']$$

The right hand side is positive if V'', $B''<0$. Q.E.D.

For this proposition, the assumptions (A1) and (A2) are not needed.

In this note, I have shown how the optimal insurance coverage of a term life insurance purchaser depends on the human wealth, nonhuman wealth and on the probability of death. The framework of Babbel & Economides allows us to perform these comparative statics exercises in an economically meaningful way even when the utility function is state dependent (because k is not necessarily equal to unity).

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