Chapter III

General Equilibruim Impacts of Increased Longevity

I. Introduction

Uncertainty regarding ones lifetime is a fundamental fact of life. So is the continual increase in the lifetimes of individuals in virtually all the countries of the world in the past few decades. What are the economic implications of such an increase (other things remaining unchanged)?

Most economists ignore the uncertainty in lifetimes of the agents in the models of intertemporal allocation of resources. The classical work by Modigliani and Brumberg (1954) ignored uncertainty regarding lifetimes of the individuals. Davies (1981) has shown that such an omission produces a downward bias in the estimation of saving and capital accumulation. In this paper, we extend the framework of Davies to endogenize the factor returns and a market for annuities to study the impact of increased longevity (and survival probabilities).

The effects of increased longevity can be analyzed as follows: First, increased longevity will affect the behavior of saving in the economy. This will in turn alter the capital accumulation in the long run through changes in the wage rate and real interest rate. Thus, the welfare of the individuals in the steady state will be affected.

To get a clear answer, we make a number of simplifying assumptions regarding the economy. These assumptions are spelled out in section two. Section three discusses the impact of a change in the distribution of survival probabilities in the population on the steady state capital accumulation and the welfare in the economy. The final section contrasts our results with the existing literature and indicates the directions for future research.

2. The Model

We carry out the analysis in the framework of Diamond (1965). However, we extend the model to incorporate uncertain lifetimes. The economy is specified as follows:

(a) **The population**: At each discrete date t ($t \geq 1$), a new generation appears; we call them generation t. Each member of generation t is alive at t, but she faces uncertainty at the beginning of period t+1: she dies with probability 1-p ($0 < p < 1$) at the beginning of period t+1. However, if an individual of generation t does not die at the beginning of period t+1, she does not die until the end of period t+1. At the end of period t+1, all surviving members of generation t face certain death. Before an individual of generation t faces death at the beginning of period t+1 she gives birth to another individual. Thus, if there are N identical individuals born at time t, then there are N+Np individuals alive at every date t.

(b) **Preference**: each individual of generation t maximizes a von-Neumann Morgenstern utility function

$$W(c_t, c_{t+1}, p) = u(c_t) + pv(c_{t+1})$$

where c_t, c_{t+1} are the consumption bundles of the individual of generation t in period t and t+1 respectively.

(c) **Technology/Annuity**: There is an available technology which converts t-good into (t+1)-good as follows: let y be the output per unit of labor and k be the capital per unit of labor. Then

$$y = f(k) \text{ with } f'(x) > 0$$
$$\text{and } f''(x) < 0 \text{ for all } x > 0.$$

We assume that each member of generation t works during t and buys a contract from the firm (with the saving), which acts as a pension fund. Therefore, the individuals of generation t get a return on their investment only if they are alive to enjoy the benefits during t+1.

(d) **Endowments**: each member of generation t is endowed with one unit of labor at t. If an individual of generation t is alive at t+1, she cannot work during t+1.

(e) **Market Structure**: We assume that the markets for labor and capital/annuity are perfectly competitive. Therefore, the competitive wage w is such that

$$w(k) = f(k) - kf'(k)$$

and the gross rate of return to capital R is such that

$$R(k) = 1 + f'(k).$$

Given that the competitive contract specifies the return only for the surviving members of every generation, an individual of generation t gets a gross rate of return R/p in period t+1 per unit saving in period t. Now we have completely described the economy.

In a steady state, a newborn individual solves the following problem:

(P) $$\underset{\{0 \le k \le w\}}{\text{maximize}}\ u(w-k) + p\, v(Rk/p)$$

She treats w and P parametrically. For the existence of a unique solution k* to the problem (P), we postulate the following assumption:

Assumption (A1): $u'(x), v'(x) > 0$ and $u''(x), v''(x) < 0$ for every $x>0$;

$$\lim_{x \to 0} u'(x) = \lim_{x \to 0} v'(x) = \infty \text{ and}$$

$$\lim_{x \to \infty} u'(x) = \lim_{x \to \infty} v'(x) = 0$$

Under the assumption (A1), the problem (P) has a unique positive solution k*.

3. The Effects of Increased Longevity

With the model described above, we can pose the questions of section one precisely: (a) how will a change in p affect k*? (b) how will a change in p affect W?

Let us first note that the answers are not obvious. A change in p will alter the saving behavior of an individual in the steady state which will in turn alter the capital accumulation pattern in the economy. Therefore, the wage rate and the real interest rate will alter too.

First, to determine the sign dk*/dp, we totally differentiate the first order condition (of the problem (P))

$$u'(w^*-k^*) = R^* v'\ (R^* k/p)$$

where

$$w^* = f(k^*) - k^*\ f'(k^*)$$

$$\text{and} \quad R^* = 1 + f'(k^*)$$

with respect to p, to get the following expression

$$dk^*/dp = A/D$$

where

$$A = R^{*2}\ k^*\ v''/p2$$

$$\text{and} \quad D = u'' + k^*\ f''\ u'' + R^*\ k^*\ f''\ v''/p + R^{*2}\ v''/p + v''f''$$

Unfortunately, it is not possible to determine the sign of D in general because it contains two positive and three negative terms. (Unless, of course, we assume that the equilibrium capital intensity is "stable," which imposes a condition on the equilibrium value and not on the environment of the economy). The other recourse is to specify forms of the utility function and the technology and calculate the changes in the value of k due to small changes in the values of p. We did this experiment with the following functional forms of the technology and the utility functions:

$$f(x) = x^{\alpha} \quad \text{where } 0 < \alpha < 1$$

and $$u(x) = x^{1-\beta}/(1-\beta) = v(x)$$

where $\beta > 1$.

For the following range of values of β = 2, 3, 4, 5 and α = .1, .2, .3, .4, .5 and base p = .1, .2, .3, ..., .9 with "small" changes in p of the order .1, <u>The changes in p and the changes in k were of the same sign</u>. Thus, for these examples, an increase in the survival probability leads to a higher capital intensity. The following table gives typical simulation results obtained:

Table 1: Value of capital intensity (k)

	p=.5	p=.6	p=.7
β=2	.05441	.07208	.08948
β=3	.03643	.05260	.06920

The value of α for the above calculations is .3

Given that a change in p is positively associated with a change in k^*, we can establish the following:

Proposition: Suppose $dk^*/dp > 0$ and $v(0) > 0$. Then, a rise in the survival probability will lead to a rise in the welfare of the individual in the steady state, i.e., $dW/dp > 0$.

Proof:

$$dW/dp = [\partial w^*/\partial p - dk^*/dp].u' + v$$

$$+ [-R^*k^*/p2 + (k^*/p)\partial R^*/\partial p + (R^*/p)dk^*/dp].v'$$

$$= v(x) - xv'(x) + k^*f'[v' - u']dk^*/dp,$$

where $x = R^*k^*/p$.

Now, from the first order condition of the optimizing problem (P), $v' - u' = -f'v'$. Therefore, by hypothesis of the proposition, the second term is positive since $f'' < 0$. By the mean value theorem, $[v(x) - v(0)]/[x - 0] = v'(y)$ for some $0 < y < x$. Therefore, $v(x) = v(0) + xv'(y)$ and $v(x) - xv'(x) = v(0) + x[v'(y) - v'(x)]$. But, $0 < y < x$ and $v'' < 0$ by (A1). Thus, $v'(y) - v'(x) > 0$ provided the first term on the right hand side of dW/dp is positive. Q.E.D.

In the discussion above, we have used the terms "a change in longevity" and "a change in the distribution of survival probability" interchangeably. This was possibly because in the two period model expected length of life and survival probability have a linear relationship (because expected length of life is, by definition,

$1\cdot(1-p) + 2\cdot p = 1+p$).

We have restricted our discussion to the case where the gross rate of return on annuities is R/p, i.e., the annuities market is actuarially fair. Do the results above change if the annuities market is <u>not</u> actuarially fair? For these examples, the answer is negative. I have performed additional simulations with the assumption that the gross rate of return on annuities is $R/\delta p$, where $\delta > 0$. The qualitative results have not changed. By some additional algebra, we can also show that the result of the proposition also remain unchanged for the actuarially unfair annuities market.

4. Relationship to the Literature and Conclusion

There seems to be very little in the literature which discuss the economic effects of changes in the lifespan. Abel (1985) lays out a model somewhat similar ours: the structure is of overlapping generations with uncertain lifetimes and perfect annuity market. However, his model contains a bequest motive for the individuals but no production. Moreover, his emphasis is on the effects of social security in that model and not on the changes in the survival probability. Kotlikoff (1979) seems to be the only other study in the literature directly dealing with the economic effects of longevity. He studies the question effect of an expanded lifespan on the capital accumulation in an economy. He also shows that for specific functional forms of utility and production, and for a range of parametric specifications, the capital intensity increases with increased lifespan. However, in his model there is no uncertainly in the life of the agents and consequently no role of annuity in the model. The phrase "expanded lifespan" in his model is interpreted as an increase in the total length of (certain) lifetime together with an increase in the number of working years. Moreover, he does not address the issue of changes in welfare due to the change in the lifespan of the agents.

We need to extend the result of the model in several directions. The first (and obvious) direction is to incorporate more general classes of utility and production functions. A non-trivial extension would involve incorporation of bequest motives of the individuals. A number of other features of a typical growth model are also absent in our formulation such as population growth and technological progress. It would be interesting to measure the qualitative and quantitative impacts of these extensions in our model.

References

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