Chapter IV

On the Demand for Annuities

1. Introduction

Uncertain lifetimes induce two different types of insurance markets: (a) annuity, and (b) life insurance. The annuity market differs crucially from the life insurance market in two important respects: (1) The demand for (term) life insurance will be generated by a bequest motive; (2) the demand for life insurance will be positive if the decision-maker (dm) chooses to borrow against future income streams. On the other hand, intertemporal decision making with uncertain lifetimes alone is a strong enough motivation for the existence of an annuity market.

In this paper, we shall investigate how survival probabilities (and longevity), transactions cost and attitude towards risk affect the demand for annuities. Some of the results are counter-intuitive. These results need not carry over to the life insurance market because of the fundamental differences between an annuity and the life insurance. The paper is organized as follows: section 2 lays out the basic two period framework in which the analysis is carried out in section 3. Section 4 concludes with a discussion of our results and directions for future research.

2. Model

The lifetime of the dm is divided into two (discrete) periods. The dm has a positive probability 1-p ($0 < p < 1$) of dying at the end of the first period. Let c_i (≥ 0) be the ith period consumption of the dm ($i=1,2$). We assume that the dm maximizes a von Neumann Morgenstern utility function of the form

$$u(c_1) + pv(\tilde{c}_2). \qquad (1)$$

Note that u is not necessarily equal to v indicating a possible state dependent utility function. In order to isolate the problem we shall assume that the dm receives a certain endowment of w in the first period and none in the second. Thus, the only uncertainty in the model comes from the lifetime uncertainty of the dm.

To specify the budget constraints we need to indicate different means by which the dm can transfer wealth between periods. We assume that there is a one-for-one storage (i.e., one unit saved in period one remains one unit in the second period regardless whether the dm is alive or dead). The other means of transferring wealth is by a purchase of an annuity contract. An annuity contract pays a predetermined amount in the second period if the dm survives through the end of period one, and it pays nothing of the dm dies at the end of period one. Therefore, the amounts contributed by all annuitants in period one is amortized among those who survive through period two (a fraction p of those who bought the annuity contracts survive). Thus, in the absence of any transactions cost, a purchase of a \$1 annuity yields \$1/p under perfectly competitive markets. For example, if everyone has a probability of survival of 0.5, an annuity purchase of \$1 would be worth \$2 in period two for those who survive because half of the individuals have died and have no claim over the annuity payment in period two. Define R as the gross rate of return on annuity. Then, it is clear from the previous discussion that

$$R = 1/kp \qquad (2)$$

where k is the proportional loading factor. The value of k depends, in part, on the cost of administration (or expense loading) and in part the degree of departure from a perfectly competitive market. If $k = 1$, the annuity market is deemed to be actuarially fair. A standard practice in the insurance literature is to assume $k > 1$. (See Fischer (1973, p. 134), for example). Lynch (1967) has shown that for term life insurance, k is indeed greater than unity. The existence of one-for-one storage in addition to annuity contracts generates an interesting question: will the dm use both the annuity and the storage to transfer wealth between the two periods? The answer is negative except for the borderline case of $k = 1/p$. In the absence of any bequest motive, the sole concern of the dm is the (risk adjusted) rates of return of various assets. If $k > 1/p$, then from (2) we get $R < 1$. Therefore, the dm will get a higher rate of return from storage. Thus, for $k > 1/p$, no annuity will be purchased. Similarly, for $k < 1/p$, no storage will be used. For $k = 1/p$, the dm will be indifferent between storage and the annuity. If the annuity is not purchased at all, a rise in the loading factor will not affect the demand for annuity contracts (the demand will be zero in all situations). Our focus here is on annuity demand. Therefore, we shall assume $k < 1/p$ for the rest of the paper.

Thus we have the budget constraints of the dm as

$$c_1 \leq w - a \qquad (3)$$

$$c_2 \leq aR. \qquad (4)$$

where a is the amount of annuity purchased by the dm.

For the results in the next section, we need a few definitions. Following Sandmo (1970), we define (intertemporal) relative risk aversion function

$$R_v(x) = -xv''(x)/v'(x) \qquad \text{for } x \geq 0.$$

We also need to make the concept of "attitute towards risk" more precise in our two-period framework. Let two dms have utility functions

$$U(c_1, c_2) = u(c1) + pu(c2) \qquad (5)$$

$$V(c_1, c_2) = v(c1) + pv(c2) \qquad (6)$$

Definition: V is said to be more risk averse than U if and only if $v(x) = f(u(x))$ for $x \geq 0$, where f is some function with $f'(x) > 0$ and $f''(x) < 0$ for $x \geq 0$.

The above definition of "more risk averse" is taken from Kihlstrom and Mirman (1974) appropriately modified for intertemporal choice. We should note that each dm has the same probability, 1 - p, of dying at the end of period one. If this probability varies across individuals, the simple comparison of them (in terms of risk aversion) will no longer be possible. We should also note that in (1) we have allowed for a state dependent utility function, but in (5) and (6) we do not.

Assumption (A1): $u', v' > 0$ and $u'', v'' < 0$ with

$$\lim_{x \to \infty} u'(x) = \lim_{x \to \infty} v'(x) = 0, \quad \lim_{x \to 0} u'(x) = \lim_{x \to 0} v'(x) = \infty$$

Given assumption (A1), the maximization of (1) subject to (2), (3) and (4) produces an interior solution for $a(=a^*$, say). The solution is uniquely characterized by the following first order condition:

$$u'(w-a^*) = v'(a^*/kp)/k \qquad (7)$$

3. Results

Using the model of section 2, we can study the effects of changes in the load factor, survival probability and the risk aversion on the optimal demand for annuity a^*. The simplest result connects the survival probability and the optimal demand for annuity.

<u>Proposition 1</u>: Under (A1), a (small) increase in the probability of survival induces the dm to buy more annuity i.e., $\partial a^*/\partial p > 0$.

<u>Proof</u>: Implicitly differentiating (5) we get

$$\frac{\partial a^*}{\partial p} = \frac{-a^* v''}{k2p(-u''-v''/pk2)}$$

which is positive by (A1) as u", v" < 0. Q.E.D.

Levhari and Mirman (1977) have shown in a general T-period model that an increase in the survival probability induces the dm to save more (they have considered the market for transferring wealth from one period to another only through saving, i.e., no anniuty market exists). However, this generality was achieved at the cost of assuming $u(c_t) = c^a{}_t/a$ for every t. Our utility function is more general; so is our market for transferring wealth. But we were unable to generalize the result for the n-period case without restricting the utility function.

<u>Remark</u> In this two period model, there is a linear relationship between expected length of life (which equals $(1-p)\cdot 1 + p\cdot 2 = p+1$) and the survival probability p. Thus, a change in the expected length of life is equivalent to a change in the survival probability. Thus, we can reinterpret Proposition 1 by saying that it represents the change in optimal annuity purchase due to a change in expected life.

Williams (1985) writes "The author believes that . . . . longer lifetimes <u>decreases</u> the demand for life annuities." (emphasis added). This seems to contradict proposition 1. To understand this seeming inconsistency, we need to take a closer look at the (implicit) model Williams has used. With a number of numerical examples he shows that the rate of return on annuity investment decreases as the longevity increases (Table 2). In our two period model, it amounts to saying that if p increases then R decreases (which is evident from equation (2)). In the limit, when p rises above 1/k, the individual will simply choose storage instead of buying annuity contracts. The discussion in Williams's paper clearly revolves around the rate of return on annuity contracts. This point is clear when he writes "If the human beings were to live forever, the annual annuity income would be the same as the annual interest on a $1000 investment. The demand for life annuity will completely disappear." In our model, the utility function contains a term involving the survival probability. In Williams's implicit model, the probability of survival enters only as a "price" of the annuity contract. Thus, his conclusion in simple economic terms is that if the price of a (normal) good increases (which will be true for annuity contracts, as the survival probability increases), then less of it will be purchased. Since our model puts additional weight to survival probablity (through its inclusion in the utility function), it is not surprising that we arrive at a different conclusion.

Proposition 2: Under (A1), a rise in the load factor k leads to an increase (decrease) in the optimal annuity purchase if the relative risk aversion R_v is greater (less) than unity, i.e.,

$$\partial a^*/\partial k > 0 \; (< 0) \text{ if } R_v > 1 \; (< 1).$$

Proof: Differentiating (7) we get:

$$[-u'' - v''/kp]\,\partial a^*/\partial k = -v' - v''(x)x = A \text{ (say)},$$

where $x = a^*/kp$. Then,

$$A = -v'(x) - v''(x)x$$

$$= -v'(x) + R_v(x)v'(x) = v'(x)\,[R_v(x)-1].$$

Thus, $A > 0$ (<0) if and only if $R_v(x) > 1$ (<1), by A1).

Q.E.D.

Proposition 2 illustrates the importance of the relative risk aversion function on the annuity purchase. An increase in transactions cost or expense loading (say, by added industry regulation) does not necessarily lead to a lower or higher demand for annuity. Note also that this result is mathematically equivalent of the response of saving due to a change in the intertemporal interest rate (Katz and Paroush (1981)).

Let a^*_u and a^*_v be the optimal choices of annuity purchases by the individual with utility functions (5) and (6) respectively when they are faced with the constraints (2), (3), and (4). How do a^*_u and a^*_v compare? We show that it depends crucially on the loading factor k in the following way:

Theorem 1: a^*_u smaller, equal or larger than a^*_v as k is greater, equal or less than unity.

Proof: The optimal first order condition for optimization can be written as:

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} = \frac{1}{k} = \frac{v'(w-a^*_v)}{v'(a^*_v/pk)} \qquad (8)$$

First, let k = 1. Then by (A1),

$$w-a^*_u = a^*_u/p, \quad \text{i.e.,} \quad a^*_u = w/(1+1/p)$$

and $w-a^*_v = a^*_v/p$, i.e., $a^*_v = w/(1+1/p)$.

That is, $a^*_v = a^*_u$.

Let $k > 1$. Then by (8) $v'(a^*_v/pk) > v'(w-a^*_v)$, i.e.,

$a^*_v/pk < w-a^*_v$ or, $u'(a^*_v/pk) > u'(w-a^*_v)$ (because u", v" < 0). Therefore $f'(u'(a^*_v/pk)) < f'(u'(w-a^*_v))$, i.e.,

$$1 < f'(u'(a^*_v/pk))/f'(u'(w-a^*_v)). \quad (9)$$

From (8) we have

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} = \frac{f'(u'(w-a^*_v)) \,.\, u'(w-a^*_v)}{f'(u'(a^*_v/pk)) \,.\, u'(a^*_v/pk)} \quad (10)$$

By using (9) in (10) we get

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} < \frac{u'(w-a^*_v)}{u'(a^*_v/pk)} \quad (11)$$

Suppose now $a^*_u \geq a^*_v$. Then

$$u'(a^*_u)pk) \leq u'(a^*_v/pk) \quad (12)$$

and

$$u'(w-a^*_u) \geq u'(w-a^*_v) \quad (13)$$

Combining (12) and (13) we get

$$\frac{u'(a^*_v/pk)}{u'(a^*_u/pk)} \geq 1 \geq \frac{u'(w-a^*_v)}{u'(w-a^*_u)} \quad (14)$$

Rewriting (14),

$$\frac{u'(w-a^*_u)}{u'(a^*_u/pk)} \geq \frac{u'(w-a^*_v)}{u'(a^*_v/pk)} \quad (15)$$

But (15) contradicts (11). Therefore $a^*_u < a^*_v$.

Similarly, we can show that if $k < 1$ then $a^*_u > a^*_v$.

Q.E.D.

To see the importance of Theorem 1 and to put the result in perspective, it is useful to examine how such questions have been dealt with in the literature. Hakansson (1969) shows that for a class of utility functions($u(x) = v(x) = x^a/a$) and for less than actuarially fair market (i.e., $k > 1$), higher risk aversion (as measured by a) leads to a lower demand for annuities. However, for $k = 1$ and $k < 1$, he writes as follows ". . . . it is not necessarily true that the more risk aversive the individual is, the more he will favor present consumption at the expense of future consumption"(p. 457). More "present consumption" translates to lower annuity demand in our model. Theorem 1 resolves the ambiguity referred to by Hakansson for the two period model. The class of utility functions he considers are the constant relative risk aversion type. In terms of our model it means that R_v (defined in section 2) is independent of x. Therefore, Theorem 1 extends Hakansson's result to a far more general class of utility functions. However, the generality of Hakansson's result has been achieved at the cost of restricting the model to two periods.

4. Conclusion

This paper determines a change in the demand for annuities due to exogenous changes in the transactions cost for an annuities, survival probabilities and attitudes towards risk. We have interpreted a change in k, a proportional loading factor, as a change in the transactions cost. We should emphasize the fact that all the comparative statics results arise from the subjective belief of the decision-maker (dm). In other words, Propositions 1 and 2 will hold as long as the dm believes that p(or k) has changed regardless whether p (or k) has actually changed or not.

Future research in this area should tackle two important questions: Do these results generalize (1) if the individuals have bequest motives? (2) if the decisions are made over more than two periods? Our conjecture is that such generalizations would require additional restrictions on the utility functions.

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