SOCIAL SECURITY REFORM, INCOME DISTRIBUTION, FISCAL POLICY AND CAPITAL ACCUMULATION

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This paper explores how a transition from *a pay-as you-go* (PAYG) social security system to a *fully-funded* (FF) one may affect income distribution , poverty, fiscal policy, and capital accumulation. With that purpose, I develop an overlapping generations, general equilibrium model with heterogeneous agents. Specifically, I argue that the transition to a FF system may have two different and opposite effects on poor workers. It makes them worse off because the FF scheme is, in contrast to the PAYG, no longer redistributive. But the transition also gives poor agents access to the financial system and the possibility of earning interest payments on their savings.

1. Introduction

This paper explores the effects that a transition from a Pay-as-you-go (PAYG) social security system to a Fully-funded (FF) one may have on income distribution, fiscal policy and capital accumulation.

The existing theoretical literature arrives at the conclusion that a FF system leads to higher steady state levels of physical capital than a PAYG. (See, for example, Arrau [1990]; Cifuentes [1996] Gonzalez [1996] and Kotlikoff [1996].) Corsetti and Schmidt-Hebbel [1995] -using an endogenous-growth model- argue that the adoption of a FF system leads to a higher level of capital not only because its funded contributions but also because it will encourage workers to become formal.

In general, the starting points in the social security literature are the Diamond [1965] overlapping generations model and the Samuelson [1958] model. One conclusion that can be inferred from these models is that a sufficient condition for the transition to increase the steady state level of capital and the savings rate of the representative agent is the real interest rate being higher than the population growth rate. A large part of the literature dealing with PAYG-FF transitions concentrates in the representative agent case.

In this paper, we introduce an overlapping generations model that departs from the representative agent assumption and use simulations to draw some conclusions for the Mexican case[1]. It is important to note, however, that this model is suitable to study social security reforms in general; it can be calibrated to resemble different countries.

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[1] In 1992, Mexico started a reform process of its social security system that will mainly consist in adopting a privately-managed fully funded system.

Two issues motivate this departure from the representative agent assumption. The first is that one of the main concerns, both inside and outside academia, is that this kind of pension reform may have negative effects on income distribution. (See for example, Diamond [1977]). The reason for this concern is simple: in many countries, PAYG social security systems include strong redistributional features in their design. For example, in the United States this is one of the main reasons why authorities do not seriously consider the option of reforming the system[2], even though this one is facing severe financial constraints. In fact, in most countries the original idea of introducing a social security system arises from a mainly distributional concern: to protect the poor old. Therefore, one way in which the agents in our model may differ is on their income levels. Specifically, these inequalities are a consequence of agents having different endowments of human capital. The second issue arises from the fact that there is evidence that, at least in some developing countries, some poor people have demand for savings but they do not have access to the financial system because their wealth is low (for example banks often require minimum deposits to open an account or some towns do not even have financial institutions.) But still their demand for savings may be a high percentage of their income. If the fraction of the population with these characteristics is high, introducing these people to the capital accumulation process may have important effects in the aggregate. In this paper we argue that, in countries with underdeveloped financial systems, a PAYG-FF pension reform may, indeed, improve the access of a broad class of citizens to financial markets.

In the case of Mexico, for example, some poor people do not have the option of channeling their savings through the formal financial system because many institutions require large initial deposits to open accounts. Also, there are substantial penalties for maintaining low balances, sometimes resulting in

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[2] Several studies,[Burkhauser and Warlick, 1981; Ferrara and Lott, 1985; Boskin et. al., 1986; Rofman, 1993; Wolf, 1987] conclude that the United States social security system presents a clear

negative real interest rates. Or, in some rural areas, banks or other financial institutions are nonexistent. Consequently, many agents who want to save part of their current income have to use informal ways to channel their savings. Mansell-Carstens [1995] shows plenty of evidence of this phenomenon for the Mexican case. She cites cases where workers ask their supervisors to retain part of their salaries as a way of safely maintaining their savings - the places where they live are not always safe to store cash. Another important way in which poor people save is through lending money to relatives and friends at low but positive interest rates. In a survey realized by Mansell-Carstens (op-cit.) with poor domestic workers, 56 percent of them revealed making a loan in the previous six months to relatives or friends. Also, some workers decide to save by buying consumer durables such as jewelry and electrical appliances. In the case of poor peasants, Mansell-Carstens reports that a common savings method is that of acquiring chicken and goats; some reported that these animals are superior as saving instruments than others such as cows and horses because they are more liquid: "you cannot sell one part of a cow when you need a little money" stated one of her informants. As already mentioned, our thesis is that many poor workers will obtain access to the formal financial system through the privatized social security system. In the Mexican case, this thesis is supported by the evidence: Mansell-Carstens (op-cit.) reports that many workers obtained their first banking account via the new system of individualized accounts that was set-up in order to carry-out the reform.[3]

It is important to note that the introduction of an obligatory FF system will give these people access to the financial system if, like in the Chilean and Mexican cases[4], individuals are forced to deposit a given amount on their private accounts, but are allowed to deposit more if they want to.

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progressive income redistribution.

[3] Although the reformed funded system was fully enforced in 1997, the system of individual accounts was introduced in 1992.

[4] For a complete description of the Chilean reform, see Diamond and Valdes-Prieto (1994). For

Thus, the second way in which agents in our model differ is that, under the PAYG equilibrium, poor agents (those endowed with less human capital) do not have access to the financial system. This means that they do not receive interest on their savings and therefore participate in the capital accumulation process of the economy.

This paper studies the effects that the transition to a capitalized retirement system may have on income distribution, capital accumulation and fiscal policy under the features described above. Section 2 introduces the model in which agents live for 2 periods and shows simulations that fit the Mexican case. In section 3 we study the transition path between the PAYG and the FF equilibria. Section 4 concludes. The annex contains an extension of the model in which agents live for 55 periods.

2. A Simple Version of the Model

This is an overlapping generations model with 2 representative agents in each generation, and in which 2 generations coexist at every period of time. The agents in this model differ in two ways: they have different endowments of human capital, and in this particular case, one type (type 2) does not have access to the financial system. The model is used to show the effects on capital accumulation, income distribution and fiscal policy, when the social security system changes from a PAYG to a FF scheme. We also assume that individuals only work during the first period of their lives and that population grows at a constant rate n. For simplicity, we assume that the stock of human capital stays constant over time.

the Mexican case, see Sales et. al. (1996).

2.1 The economy under the PAYG system:

The production function is of the form:

$$Y_t = K_t^a (H^1 L_t^1 + H^2 L_t^2)^{(1-a)} \qquad \text{(1)}$$

Where H^i is the stock of human capital owned by agents of type i (constant over time for simplicity) and L_t^i is the fraction of the population of type i alive at time t. K_t is the stock of physical capital. Also, $0 < \alpha < 1$.

In this economy, aggregate production takes place using physical capital and two types of labor, provided by the two different types of agents. As we mentioned before, each type of agent has a different endowment of human capital; that is, they have different types of skills that result in different productivities. Accordingly, agents of type i provide, in period t, $H^i L_t^i$ of the total labor input.

Total population is the summation of agents of types 1 and 2. Of this total, a fraction b, where $0 < b < 1$, is of type 1 and (1−b) of type 2:

$$L_t = L_t^1 + L_t^2 \qquad \text{(2)}$$

$$L_t^1 = bL_t \qquad \text{and} \qquad L_t^2 = (1-b)L_t \qquad \text{(2')}$$

Population growth rate is given by n:

$$L_t = (1+n)L_{t-1} \qquad \text{(3)}$$

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Agents of type 1 own γ of the total stock of human capital and agents of type 2 the rest. One way to think about this is that there is a total stock of knowledge and that workers only have partial access to it. Alternatively, we can say that there are two different sets of skills or two different forms in which workers can participate in the production of an aggregate homogeneous good. We assume that agents of type 1 have a larger endowment of human capital, and therefore higher labor incomes, than agents of type 2. We will denote agents of type 1 as rich agents and agents of type 2 as poor agents.

$$H = H^1 + H^2 \tag{4}$$

$$H^1 = gH \qquad \text{and} \qquad H^2 = (1-g)H \qquad (\gamma > 0.5) \tag{4'}$$

We can interpret equations (2') and (4') as if they meant that there are two representative agents in the economy and that each of them owns a fraction of the economy's total endowment of units of effective labor $(H^1 L_t^1 + H^2 L_t^2)$ that is proportional to their respective shares of labor and human capital. In other words, these equations are equivalent to saying that agents of type 1 own a fraction q of the economy's total endowment of units of effective labor, where

$$q = \frac{gb}{2gb + 1 - g - b} \tag{5}$$

It should be noted that different compositions of the population and different distributions of the stock of human capital translate into different levels of effective labor and, therefore, into different output levels. This should be obvious: if there are two economies that have the same stock of human capital

and the same population size, the one in which a higher fraction of its population has a higher access to the stock of human capital will produce more.

Each generation has two representative agents each with logarithmic utility function of the form:

$$U_t^i = \ln C_{y,t}^i + \frac{1}{(1+r)} \ln C_{o,t+1}^i \qquad\qquad i=1,2 \qquad (6)$$

Where $C_{y,t}^i$ is consumption of agent i when young at time t and $C_{o,t+1}^i$ his consumption when old at time $t+1$.

Second period consumption for agent 1 is given by:

$$C_{o,t+1}^1 = (1+r_{t+1})\left[(1-t_s)H^1 w_t - C_{y,t}^1\right] + \left[t_s\, j\, Hw_t(1+n)\right] \qquad (7)$$

and second period consumption for agent 2 is:

$$C_{o,t+1}^2 = \left[(1-t_s)H^2 w_t - C_{y,t}^2\right] + \left[t_s\, j\, Hw_t(1+n)\right] \qquad (8)$$

where:

$$j = gb + (1-g)(1-b) \qquad (9)$$

The wage rate per unit of raw labor is given by w; that is, each worker's labor income[5] is this wage rate enhanced by its productivity. t_s is the social

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[5] Income for workers is given by their marginal productivity. So labor income for each type of agent is given by:

security contribution rate. Equations (7) and (8) imply that the system is strongly redistributive; it taxes both individuals at the same rate and uses the revenues to pay equal pensions to current old individuals of both types. This result can be verified by dividing the total value of contributions to social security in one period by the labor force size of the previous period:

$$\frac{\tau_s w_t (H^1 L_t^1 + H^2 L_t^2)}{L_{t-1}} = \tau_s \varphi H \, w_t \, (1+n)$$

This is the pension that all old individuals, no matter their type, receive at period t. Although a PAYG system does not necessarily has to be redistributive, this is the case in many social security systems in the world. In most cases, the initial idea of introducing a social security system came from the desire to redistribute income in favor of the poor and old individuals. In Latin America, for example, Argentina, Brazil, Venezuela, Colombia and in Chile before the early 80s reform, the PAYG social security systems included important redistribution schemes. (See for example Barreto de Oliveria [1994] and Vittas [1997]). In Europe, Switzerland and, to a lesser extent, Germany also have PAYG social security systems that are clearly redistributive. (Queisser 1996).

It should also be noted that, contrary to models with homogeneous agents, the implicit rate of return of the PAYG system is different from the population growth rate. This rate will be higher than n for poor individuals (individuals of type 2 in our model) and lower than n for rich individuals. Note that equation (8) reflects the fact that poor individuals' savings do not have

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$$\frac{\partial Y_t}{\partial L_t^i} = H^i \, (1-\alpha) \, k_t^\alpha, \text{ where } k_t = \frac{K_t}{H^1 L_t^1 + H^2 L_t^2} \, .$$

access to the financial system and therefore they do not receive interest payments.

Each agent maximizes (6) subject to their respective budget constraint (implicit in equations (7) and (8)). From the first order conditions we obtain consumption in each period for both types:

$$C^1_{y,t} = \frac{1+r}{2+r}\left[(1-t_s)H^1 w_t + \frac{t_j\, Hw_t(1+n)}{(1+r_{t+1})}\right]$$
(10)

$$C^2_{y,t} = \frac{1+r}{2+r}\left[(1-t_s)H^2 w_t + t_j\, Hw_t(1+n)\right]$$
(11)

So that savings for each individual are given by:

$$S^1_t = (1-t_s)H^1 w_t - \frac{1+r}{2+r}\left[(1-t_s)H^1 w_t + \frac{t_j\, Hw_t(1+n)}{(1+r_{t+1})}\right]$$
(12)

$$S^2_t = (1-t_s)H^2 w_t - \frac{1+r}{2+r}\left[(1-t_s)H^2 w_t + t_j\, Hw_t(1+n)\right]$$
(13)

The equilibrium condition for the economy is:

$$K_{t+1} = S^1_t L^1_t$$
(14)

That is, only the savings that are channeled through the financial system are accumulated as capital and take part in the economy's production process. It should also be noted that contributions to the social security system are not saved as capital; they are directly paid as pensions to current old people. This is

the key feature of a PAYG system: it is a transfer system between different generations.

Combining equations (12) and (14) we can obtain an equation describing the evolution of capital from period *t* to period *t+1*.

$$K_{t+1} = \frac{1}{2+r}\left[(1-t_s)H^1L_t^1w_t - \frac{1+r}{2+r}\frac{t_s j\, HL_t^1 w_t(1+n)}{(1+r_{t+1})}\right] \qquad \textbf{(15)}$$

Expressing (15) in units of effective labor by dividing both sides by $(H^1L_t^1 + H^2L_t^2)$ yields:

$$K_{t+1} = \frac{1}{1+n}w_t\left[(1-t_s)q\frac{1}{2+r} - \frac{1+r}{2+r}\frac{t_s b(1+n)}{(1+r_{t+1})}\right] \qquad \textbf{(16)}$$

where $k_t = \dfrac{K_t}{H^1L_t^1 + H^2L_t^2}$ is the stock of capital per unit of effective labor. Note that because the stock of human capital stays constant over time, the units of effective labor also grow at rate *n*.

The payments to factors are given by:

$$w_t = (1-a)k_t^a \qquad \textbf{(17)}$$

$$r_t = a k_t^{(a-1)} \qquad \textbf{(18)}$$

Using these results, an equation that implicitly describes the evolution of capital can be obtained:

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$$K_{t+1} = \frac{1}{1+n}(1-a)k_t^a \left[(1-t_s)q\frac{1}{2+r} - \frac{1+r}{2+r}\frac{t_s b(1+n)}{(1+ak_t^{(a-1)})} \right] \qquad \textbf{(19)}$$

Finally, the following equation shows k_{PG}, the PAYG steady state value of capital per unit of effective labor in an implicit form.

$$K_{PG}^{(1-a)} = \frac{(1-a)}{1+n}\left[(1-t_s)q\frac{1}{2+r} - \frac{1+r}{2+r}\frac{t_s b(1+n)}{(1+ak_{PGt}^{(a-1)})} \right] \qquad \textbf{(20)}$$

2.2 The Fully-Funded System

When the economy is at this steady state, the government introduces the FF system. Under it, individuals have their own individual accounts in which they deposit their contributions to the social security. They are still obligated to deposit at least a fraction t_s of their labor income although they can deposit more. When they become old and retire, they receive their total contributions plus an interest payment (pensions' interest rate is equal to the rate of return on capital). Now the amount of each individual's pension is only determined by its own resources.

Individuals of type 2 that did not have access to the financial system in the old scheme now have access through then new pension system. A priori, we can expect that the introduction of this system will have three different effects on the income of poor individuals (individuals of type 2): first they will be worse off because the new system is no longer redistributive; second, they will better off because the new system gives them access to the financial system so that now they will receive interest payments on its savings, and finally, they may benefit from changes in the real payments to factors. On the other hand, we expect to see

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an increase in the stock of physical capital due to two factors: first, as we explain below, the design of the funded system implies an increase in capital accumulation and second, our assumption that the reform will provide access to the financial system to poor individuals means that their savings will also be accumulated as capital.

Now the agents face different budget constraints than in the PAYG case:

$$C^i_{o,t+1} = (1+r_{t+1})\left[(1-t_s)H^i w_t - C^i_{y,t}\right] + (1+r_{t+1})t_s H^i w_t \qquad \text{for i=1,2} \qquad \textbf{(21)}$$

Notice that now the pension that individuals receive when old depends only on their own contributions to the system. The pensions are equal to these contributions plus interest payments.

Solving the utility maximization problem, we get consumption in period 1 for both types:

$$C^i_{y,t} = \frac{1+r}{2+r} H^i w_t \qquad \text{for i=1,2} \qquad \textbf{(22)}$$

and savings for each individual are:

$$S^i_t = (1-t_s)H^i w_t - \frac{1+r}{2+r} H^i w_t \qquad \text{for i=1,2} \qquad \textbf{(23)}$$

Total savings in the economy are:

$$S_t = L^1_t S^1_t + L^2_t S^2_t \qquad \textbf{(24)}$$

And the new the equilibrium condition for the economy is:

$$K_{t+1} = S_t + D_t \qquad \qquad \textbf{(25)}$$

Where $D_t = t_s w_t (H^1 L_t^1 + H^2 L_t^2)$ is the total value of contributions made by the young at time *t*. That is, now the contributions to the social security system also form part of the capital accumulation process. Replacing (24) into (25) yields:

$$K_{t+1} = \frac{1}{2+r} w_t [H^1 L_t^1 + H^2 L_t^2] \qquad \qquad \textbf{(26)}$$

Expressing (26) in units of effective labor and replacing for the value of labor income yields the equation that describes the evolution of capital in the FF case:

$$k_{t+1} = \frac{1}{(2+r)} \frac{1}{(1+n)} (1-a) k_t^a \qquad \qquad \textbf{(27)}$$

It should be pointed out that this equation is the same for an economy without any type of social security, as long as the contributions that have to be paid to the system do not exceed the level of savings that individuals would have chosen in the absence of social security. This happens because now that social security contributions go through the financial system, individuals have the option of choosing the exact allocation of consumption between the two periods of their lives that they would have chosen in the absence of social security: if they decide to consume a higher fraction of their earnings when young, they are able to borrow against their future benefits at the market interest

rates[6]. This implies that increases in social security contribution rate will be matched with decreases in individual saving rates.

Finally, the steady state level of capital per unit of effective labor in the FF case is:

$$k_{ff} = \left[\frac{1}{(2+r)} \frac{1}{(1+n)} (1-a) \right]^{1/(1-a)}$$

(28)

2.3 Comparing the Steady State under both Systems

In order to obtain a meaningful comparison between both systems, we assign realistic values to the model's parameters in order to find numerical solutions. In this particular case, we use values that resemble the case of the Mexican economy.

Table 1

Parameters in Base Case Scenario

Parameter	Value
Capital's share in production	$\alpha = .5$
Discount rate	$\rho = .03$
Population growth rate	$n = .02$
Social Security tax rate	$\tau s = .115$

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[6] Although this might not be very realistic assumption for countries with under-developed financial systems, we use it to show in a more clear way the effects of elimination of liquidity

Calibration

The share of capital in total income is represented by α. From the Mexican National Accounts, we know that this value has fluctuated around 0.5 during the last 20 years. (See Arrau 1990). In the empirical literature, we find a great variety of estimates for the discount rate, ρ. (Auerbach and Kotlikoff [1987]; Hansen and Singleton [1983]; Haussman [1979] or Hubbard et. al. [1995]). Here we choose ρ= 0.03 which is consistent with many such studies. The population in Mexico is expected to grow at an approximate 2% rate for the next 30 years[7]. And the value of the Social Security contribution rate that we choose is the one that will apply with the reform. Finally, β and γ are, respectively, the fraction of the labor force that is skilled (rich agents) and the fraction of the total stock of human capital that they own. Because of the lack of empirical estimates for these variables, we use several different possible values for them. Recall that β does not stand for the fraction of the population that is poor; it stands for the percentage of the labor force that is at the bottom part of the wage spectrum and does not have access to the financial system.

2.3.1 The Steady State level of capital

In order to find the steady state values of capital using our parameter values, we plot k_{t+1} against k_t , or the right-hand side minus the left-hand side of equations (20) and (28). This is shown in Figures 1 and 1b. Table 2 shows the effects of the reform on capital per unit of effective labor and the capital-output ratio for different combinations of β and γ. As can be expected, the effects of pension reform on the steady state level of capital are larger the larger the

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constraints.

[7] Source: US Bureau of the Census.

fraction of the population represented by agents of type 2, or the lower the endowment of human capital owned by them. It is interesting to look at the case where γ and β are both equal to one. This is the homogeneous agent case. In this particular scenario, the reform increases the steady state level of capital by 38.4% and the capital-output ratio by 17.6. This means, that even in a case where access to the financial system is unrestricted, the adoption of a funded social security system increases the long run level of physical capital. In the β = γ = 0.6 case, the stock of capital per unit of effective labor is 1.8 times larger in the FF steady state than in the PAYG one, and the capital-output ratio is 67% higher. In general, the increase in the stock of physical capital after the reform is larger the lower the fraction of the total units of effective labor owned by agents of type 2. In all cases the reform to the social security system leads to higher levels of capital per unit of effective labor and of the capital-output ratios. This happens because, in the FF case, contributions are accumulated as capital instead of being transferred to current pensioners. So even when saving rates, as fraction of wages, do not increase, the economy will reach higher levels of capital[8].

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[8] The result of the model are robust to sensitivity changes in α and ρ.

Table 2

Steady State levels of capital and capital-output under both systems

for different values of β and γ

Values of β and γ	K$_{ff}$	k$_{py}$	% change	K/Y $_{ff}$	K/Y $_{py}$	%change
β= .5, γ=.5	0.0583	0.0109	434.86	0.2414	0.1044	131.27
β=.4 , γ=.7	0.0583	0.0163	257.66	0.2414	0.1276	89.18
β=.6, γ=.6	0.0583	0.0207	181.64	0.2414	0.1438	67.87
β=.5, γ=.7	0.0583	0.0214	172.42	0.2414	0.1462	65.05
β=.5, γ=.8	0.0583	0.0280	108.21	0.2414	0.1673	44.27
β=.7 γ=.7	0.0583	0.0307	89.90	0.2414	0.1752	37.77
β= .6 , γ=.8	0.0583	0.0319	82.75	0.2414	0.1786	35.15
β=.7, γ=.8	0.0583	0.0351	66.09	0.2414	0.1873	28.85

β=.5, γ=.9	0.0583	0.0355	64.22	0.2414	0.1884	28.13
β= .6 , γ=.9	0.0583	0.0377	54.64	0.2414	0.1941	24.33
β=.8, γ=.8	0.0583	0.0379	53.82	0.2414	0.1946	24.00
β= .7 , γ=.9	0.0583	0.0393	48.34	0.2414	0.1982	21.77
β=.8, γ=.9	0.0583	0.0405	43.95	0.2414	0.2012	19.95
β=.9, γ=.9	0.0583	0.0414	40.82	0.2414	0.2034	18.68
β=1, γ=1	0.0583	0.0421	38.48	0.2414	0.2052	17.68

2.4 Income Distribution

As discussed before, one of the main concerns regarding the privatization of social security systems is that income distribution will deteriorate, in particular hurting the poor old. James (1997) argues that unless the privatization program contains explicit redistributive mechanisms, income distribution may deteriorate. Arrau and Schmidt-Hebbel (1994), in an overview of the Pension literature, point that much research is needed in order to have a more clear idea of how pension privatization affects income distribution. (For an example of this view in the United States see Leone [1997]). One of the purposes of this paper is to study this issue. In our context, it is difficult to predict a priori the effects of the reform. We saw that what happens to poor agents is uncertain and the same is true for rich agents: on the one hand they are better off because they do not have to subsidize redistributional pensions to poor agents. But on the other hand, we know that there will be changes in the relative prices of factors and that this may have negative effects on their wealth. Here we will measure income distribution as the ratio of rich agents' over poor agents' present value of lifetime incomes.

The present value of lifetime income for agents of type 1 in the PAYG steady state is: $(1-t_s)H^1 w_{PG} + t_s j\, Hw_{PG}(1+n)$ and the equivalent expression for agents of type 2 is: $(1-t_s)H^2 w_{PG} + \dfrac{t_s j\, Hw_{PG}(1+n)}{(1+r_{PG})}$ -where r_{PG} and w_{PG} are respectively the steady state levels of the interest rate and wages. With these two expressions, we can obtain the ratio of present value earnings of rich over poor individuals, which is given by:

$$\frac{(1+r_{PG})[g(1-t_s)+t_s j\,(1+n)]}{[(1-t_s)(1-g)(1+r_{PG})+t_s j\,(1+n)]}$$

From this expression, we can obtain the condition under which agents of type 1 will have higher lifetime earnings than agents of type 2. This condition is:

$$g > \frac{1}{2} - \frac{t_s j\,(1+n)r_{PG}}{2(1+r_{PG})(1-t_s)}$$

This condition means that, because of the redistributional features of the PAYG system, being endowed with a higher level of human capital $(\gamma > .5)$, is not a sufficient condition for agents of type 1 to have higher lifetime earnings than agents of type 2: income inequality is not only determined by inequalities in labor earnings, but also by capital earnings and by the design of the social security system.

In the FF case, the ratio of present value of lifetime incomes of rich over poor individuals is just:

$$\frac{g}{(1-g)}$$

That is, because the FF system is not redistributional at all, having a larger endowment of human capital is a sufficient condition to have higher lifetime earnings. So if $\gamma > 1/2$ (which was already assumed), individuals of type 1 will have higher income, in present value, than individuals of type 2.

Following this argument, the reform of the social security system will improve the distribution of income (will make it more equal) if the ratio of lifetime earnings between rich and poor agents is lower in the FF steady state than in the PAYG one.[9]

Table 3 shows the percentual change of the ratio of rich to poor agents' income for different values of β and γ. The first two columns show this ratio under the steady state in the FF and PAYG systems respectively and the third one is the percentual change between them. The different combinations of β and γ are ordered according to the size of change in income distribution after the reform to the social security system. A negative change means more income equality between agents of both types. For example the $\beta = 0.5$, $\gamma = 0.5$ case is the one in which there is more redistribution after the reform (the ratio goes from 1.2 to 1, that represents complete equality) and the $\beta = 0.9$, $\gamma = 0.9$ case is the one in which income inequality gets worst - the ratio increases by 17.27%. We can see that the redistributive effect of the reform is decreasing with γ, the fraction of the total stock of human capital in hands of individuals of type 1. This result predicts that a reform of the type described here will deteriorate income inequality if large gaps between the skills of workers exist. Also, for any given γ, a lower β will mean a more equal income structure after the reform. This is because, the lower the value of β means a higher increase in the stock of physical capital in the FF steady state and consequently, a lower interest rate; therefore, for given differences in labor earnings, the gap in capital earnings will be lower.

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[9] Because the solution for the steady state level of capital may not be unique, an analytical condition for improvements on income distribution cannot be derived.

When the share of the total stock of units of effective labor in hands of rich agents is higher than 80%, income distribution worsens after the reform.

It is interesting to give a closer look at the β=0.5 , γ=0.5 case. When agents of both types represent exactly half the population size and own half the stock of human capital, their lifetime incomes are the same in the FF steady state. They are different in the PAYG system because agents of type 2 have no capital gains. Thus this case depicts a situation in which income inequality is explained only by differences in capital income and not at all by differences in labor income. In this case, given our thesis that the reform to the social security system will provide access to the financial system, income inequality disappears after the reform.

Table 3

Ratio of Rich to Poor Individuals' Income under both Systems

for different values of β and γ

Values of β and γ	(l_1/l_2)ff	(l_1/l_2)pg	% Change
β=.5 , γ=.5	1.00	1.10	-9.68
β=.6 , γ=.6	1.50	1.61	-6.84
β=.4 , γ=.7	2.33	2.44	-4.20
β=.5 , γ=.7	2.33	2.43	-4.08
β=.7 , γ=.7	2.33	2.42	-3.90
β=.5, γ=.8	4.00	3.99	0.022
β=.6 , γ=.8	4.00	3.98	0.44
β=.7 , γ=.8	4.00	3.96	0.84
β=.8 , γ=.8	4.00	3.95	1.22
β=.5 , γ=.9	9.00	8.17	10.03
β=.6 , γ=.9	9.00	8.03	11.94
β=.7, γ=.9	9.00	7.91	13.77

β=.8 , γ=.9	9.00	7.78	15.55
β=.9 , γ=.9	9.00	7.67	17.27

2.5 *Poverty*

Apart from looking at changes in income distribution, we would like to see if the poor agents in our model are better or worse off after the transition. As we said before, there are effects going in opposite directions so we can not know the overall result a priori. Here we compare the net wealth's present value for the representative poor agent under both equilibria.

As we already noted, in the PAYG steady state, the present value of income for poor agents is : $(1-t_s)H^2 w_{PG} + \dfrac{t_s j\, Hw_{PG}(1+n)}{(1+r_{PG})}$. And the equivalent of this expression in the FF case is: $H^2 w_t$. Therefore, agents of type 2 will be better off after the transition if :

$$\frac{w_{ff}}{w_{py}} > [(1-t_s) + \frac{t_s j\ (1+n)}{(1+r_{py})(1-g)}]$$

where w_{ff} and w_{py} are the equilibrium wage rates for the FF and PAYG systems and r_{ff} is the equilibrium interest rate in the PAYG steady state. This expression says that having higher labor incomes after the transition is not a sufficient condition for an increase in lifetime income for poor agents; the access to the financial system and the loss of redistributive pensions also need to be taken into account. Table 4 shows the present value of lifetime income for

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individuals of type 2 under both systems and the percentual change between them after the transition for different values of β and γ. In all cases, poor individuals are better off after the reform. The increase in the stock of physical capital means increases labor productivity enough to improve lifetime earnings for poor individuals. Of course, this improvement is larger the larger is the fraction of the total stock of effective labor owned by individuals of type 2. Even though the effects of pension reform on social security on income distribution may be uncertain, poor individuals will be better off in absolute terms because the reform will translate into higher labor productivity.

Table 4

Poor Individuals' Income under both Systems

for different values of β and γ:

Values of β and γ	I2 pg	I2 ff	% Change
β=.5 , γ=.5	0.046	0.120	159.77
β=.6 , γ=.6	0.064	0.120	88.51
β=.4 , γ=.7	0.056	0.120	112.43
β=.5 , γ=.7	0.065	0.120	85.40
β=.7 , γ=.7	0.078	0.120	54.80
β=.5, γ=.8	0.074	0.120	62.09
β=.6 , γ=.8	0.079	0.120	51.86
β=.7 , γ=.8	0.083	0.120	44.77
β=.8 , γ=.8	0.086	0.120	39.33
β=.5 , γ=.9	0.083	0.120	43.95
β=.6 , γ=.9	0.086	0.120	39.69
β=.7, γ=.9	0.088	0.120	36.82
β=.8 , γ=.9	0.089	0.120	34.78
β=.9 , γ=.9	0.090	0.120	33.32

2.6 Complete access to the Financial System

We now consider the case in which inequality in the endowments of human capital exist but in which all agents have access to the financial system. This case will resemble a more advanced economy with a developed financial system.

When all agents have access to the financial system, the increase in the stock of capital per unit of effective labor after the reform is that of the homogenous agent case for all different values of β and γ: the savings of all individuals are channeled through the financial system. However, the implications for income inequality between agents of both types are different. Individuals of type 2 already receive interest payments on their savings in the PAYG system. After a social security reform of the type we are considering here, they will be worse off with respect to agents of type 1 because the new system will no longer be redistributive. Table 4 shows the same variables as in Table 3 for the case in which all agents have access to the financial system in the PAYG equilibrium. It can be seen that in all cases income inequality increases. Again, this inequality will be larger the higher is the fraction of the stock of units of effective labor owned by agents of type 1. In the extreme case, when $\beta = \gamma = 0.9$ the ratio of lifetime income between the two agents increases by 86%. When there is a developed financial system in which all agents have access to it, a reform to the social security of the type described here will always increase income inequality.

Table 5

Ratio of Rich to Poor Individuals' Income under both Systems

for different values of β and γ:

Complete Access to the Financial System

Values of β and γ	(l_1/l_2)ff	(l_1/l_2)pg	% Change
β=.5 , γ=.5	1.00	1.00	0.00
β=.6 , γ=.6	1.50	1.43	5.15
β=.4 , γ=.7	2.33	2.11	10.68
β=.5 , γ=.7	2.33	2.09	11.53
β=.7 , γ=.7	2.33	2.06	13.19
β=.5, γ=.8	4.00	3.25	22.95
β=.6 , γ=.8	4.00	3.19	25.47
β=.7 , γ=.8	4.00	3.13	27.95
β=.8 , γ=.8	4.00	3.07	30.38
β=.5 , γ=.9	9.00	5.81	54.87
β=.6 , γ=.9	9.00	5.52	62.96
β=.7, γ=.9	9.00	5.27	70.87
β=.8 , γ=.9	9.00	5.04	78.62
β=.9 , γ=.9	9.00	4.83	86.20

2.7 The role of Fiscal Policy in the Reform

In our model, the government is only concerned with the social security system. It only collects taxes that will be used to finance the system. So far, the only role that the government has in the PAYG steady state equilibrium is to

realize the transfers from young to old individuals, and it has no active role in the FF equilibrium. In a social security reform of this type, however, the government plays a fundamental role in the transition path from one steady state to the other. If the government has to meet the obligations acquired with transition workers, as is the case with the Mexican reform, it needs to generate revenues in order to pay pensions to these workers. This changes substantially some of the results presented above. If the government assumes this debt, the steady state that the economy reaches after the reform will be lower. This is because the capital stock is determined not only by private wealth but also by the economy's total wealth, that is private plus public. This explicit debt means a negative public wealth that crowds out the national capital stock. For example, if the authorities decide to switch to a FF system at time t, they will have to pay pensions to current old individuals that contributed to the system in period t-1, but now they will not collect contributions from current young workers; at this time young individuals are making their social security contributions to their own individual accounts. In this section, we study a reform on social security in the case in which government recognizes this debt. We introduce an income tax that will generate the government revenue necessary to honor pension obligations with transition workers. The following equation describes the government's debt position across time:

$$B_{t+1} = B_t(1+r_t) + G_t - [\mathsf{t}_s w_t(H^1 L_t^1 + H^2 L_t^2) + \mathsf{t}_t Y_t] \qquad \textbf{(29)}$$

Where B_t and G_t are government's debt and expenditures at time t respectively, and t_t is the income tax rate at period t. In the case of the unfounded system, the income tax rate is zero and government expenditures, G_t, are the pensions paid to old individuals which are equal to the contributions made by the young:

$$G_t = (1+n)t_s w_{t-1}(H^1 L_t^1 + H^2 L_t^2) = t_s w_t(H^1 L_t^1 + H^2 L_t^2)$$

This means that, because the only concern for the government in this economy is the pension system, there is no change in the level of debt, and in the PAYG steady state B_{t+1} equals B_t . (For simplicity, we assume the initial debt level equal to zero.) When the government issues debt, it crowds out private savings. (People see investments in government bonds and in private financial markets as perfect substitutes.) Therefore, equations (14) and (25) now become:

$$K_{t+1} = S_t^1 L_t^1 - B_{t+1} \qquad \textbf{(14')}$$

$$K_{t+1} = [L_t^1(S_t^1 + D_t^1) + L_t^2(S_t^2 + D_t^2)] - B_{t+1} \qquad \textbf{(25')}$$

Where D_t^i are the contributions made to the social security by an individual of type i. Again, this is the key difference between both systems. Whereas in an unfounded system social security's contributions do not take part in the capital accumulation process, in a funded system, not only people's extra savings[10] are accumulated as capital, but also are the contributions themselves. This is why a Fully-Funded social security system leads to higher levels of capital and not, as it is sometimes believed, because reforming the system creates incentives for individuals to save more. As we show here, the adoption of a FF system will mean more capital accumulation even if the savings rates, as proportion of labor incomes, do not change.

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[10] Those made in addition to the required contributions.

If the reform to a funded system happens in year t=R, government debt will be equal to the value of pensions paid to individuals that are old at time R. From equation (29):

$$B_R = t_s w_{R-1}(H^1 L_t^1 + H^2 L_t^2) \qquad (30)$$

B_R is the fiscal cost of the reform. This cost, as a fraction of the reform's year income, is equal to:

$$\frac{B_R}{Y_R} = t_s(1-a) \qquad (31)$$

In our base case scenario this cost is equal to 5.75% of the GDP in the period in which the authorities switch to a funded system. For the contribution rate prevailing in Mexico, the cost of the reform can amount up to 11.5% of GDP, depending on the value of α. Although these values of the fiscal cost may seem big, we will see in the next section that they are sub estimated because they are obtained from a two period model. If the government recognizes its debt with individuals that contributed to the system in the past, it will have to pay pensions to more than one generation and, more important, for more than one period. Obviously, if the government assumes this debt, [11] the capital accumulation gains will be lower.

As we just stated, this burden will have some crowding out effect on the economy's capital stock. Following equations (25') and (30), we can find the steady state of capital for the FF system. First, we express the evolution of government debt in per capita terms, taking into account that after the reform

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[11] In practical terms, it would be politically impossible not to recognize this debt. Although so doing reduces the stock of capital, those benefited will mostly be living generations.

government no longer collects social security contributions, that is, $t_s = 0$ for t> R.

$$b_{t+1} = \frac{1}{(1+n)}[b_t(1+r_t) + g_t + -t_t y_t] \tag{32}$$

where variables in lower case are in terms of units of effective labor. Therefore, the fiscal cost of the reform in percapita terms is:

$$b_R = \frac{t_s w_{PG}}{(1+n)} \tag{33}$$

Where w_{PG} is the wage rate at the Pay-as-you-go steady state. Also, because agents now face an income tax, the equation describing the motion of capital per unit of effective labor now becomes:

$$k_{t+1} = \frac{1}{(1+n)} \frac{1}{(2+r)} (1-a)(1-t_t) k_t^a - b_{t+1} \tag{34}$$

The government has several different alternatives to finance the transition cost. It can, for example, apply very high income rates in order to pay the cost faster, thus making current generations and those born in the near future to pay the reform's cost. Or it may choose to make generations in the long future to share part of the cost by using low income tax rates and spreading the cost during several periods. Each different form of financing the transition will have different effects on intergenerational distribution and on capital evolution. Here we will consider the following government action: imposing an income tax each period in order to maintain a constant level of government debt per unit of effective labor. This implies applying an income tax rate equal to:

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$$t_t = \frac{b_R(r_t + n)}{y_t} \qquad\qquad \text{for } t > R. \qquad\qquad \textbf{(35)}$$

Consequently, the stock of capital per unit of effective labor will evolve according to the following nonlinear difference equation:

$$k_{t+1} = \frac{1}{(1+n)}\frac{1}{(2+r)}(1-a)(1 - [b_R(ak_t^{(a-1)} + n)/k_a^t])k_t^a - b_R \qquad\qquad \textbf{(34')}$$

Table 6 shows the steady states levels of capital per unit of effective labor when the government recognizes its debt with past contributors and uses the strategy described above to finance this debt. One important change is that the steady state that the economy reaches with the FF system depends, in contrast with the previous case, on the initial level of capital, that is, on the PAYG steady state. We can see that the higher the fraction of the population that does not have access to the financial system, the higher the steady state level of capital per unit of effective labor that the economy reaches after social security privatization; that is, the economy not only reaches a higher level of capital relative to the PAYG level, but it also reaches a higher absolute level. This result strengthens the conclusion that the benefits from privatizing social security, in terms of capital accumulation are higher the higher are the imperfections in the capital markets, that in this case are reflected as parts of the population being segregated from the financial system.

Of course, the steady state levels of capital and income are lower when the debt with past contributors to the system becomes explicit. This is because government debt crowds out private savings. The effect of recognizing this debt is shown in Figure 2. We can see, for example, that in the homogeneous agent case, capital per unit of effective labor increases by 12.82% after the transition to a fully-funded system; this is substantially lower than the 38.48% increase

obtained when obligations with past workers were not assumed. Comparing Tables 2 and 6, we can see that this result holds for every combination of β and γ shown.

The table also shows that the capital-output ratios are higher after the transition, but lower than in the previous non-debt case. In the single agent case, this ratio is equal to 0.21 in the FF case: an increase of 5.85%.

Table 6

Steady State levels of capital and capital-output under both systems

for different values of β and γ:

explicit government debt

Values of β and γ	K_{ff}	K_{py}	% change	K/Y $_{ff}$	K/Y $_{py}$	%change

β= .5, γ=.5	0.0526	0.0109	382.57	0.229	0.104	119.67
β=.4 , γ=.7	0.0514	0.0163	215.34	0.227	0.128	77.58
β=.6, γ=.6	0.0506	0.0207	144.44	0.225	0.144	56.35
β=.5, γ=.7	0.0504	0.0214	135.51	0.224	0.146	53.46
β=.5, γ=.8	0.0494	0.0280	76.43	0.222	0.167	32.83
β=.7 γ=.7	0.0490	0.0307	59.61	0.221	0.175	26.34
β= .6 , γ=.8	0.0488	0.0319	52.98	0.221	0.179	23.68
β=.7, γ=.8	0.0484	0.0393	23.16	0.220	0.198	10.98
β=.5, γ=.9	0.0483	0.0355	36.06	0.220	0.188	16.64
β= .6 , γ=.9	0.0481	0.0377	27.59	0.219	0.194	12.95
β=.8, γ=.8	0.0480	0.0379	26.65	0.219	0.195	12.54
β= .7 , γ=.9	0.0478	0.0393	21.63	0.219	0.198	10.29
β=.8, γ=.9	0.0477	0.0405	17.78	0.218	0.201	8.53
β=.9, γ=.9	0.0476	0.0414	14.98	0.218	0.203	7.23
β=1, γ=1	0.0475	0.0421	12.82	0.217	0.205	5.85

Table 7 shows the value of the transition cost, b_R, as percentage of reform's year capital and output per unit of effective labor as well as the income tax rate that would be required in the FF steady state in order to keep the public debt constant in terms of units of effective labor. The transition cost is higher, as a percentage of capital per capita, the higher is the fraction of the population that has access to the financial system. On the other hand, as explained above, the cost as a fraction of the output per unit of effective labor in the year of the reform is the same for different combinations of β and γ. In the homogeneous agent case, this cost represents 27.47% of the stock of capital per unit of effective labor and 5.63% of the economy's output per unit of effective labor. This figure is, in contrast, 53.99 for the case in which poor agents own 50% of the economy's stock of human capital and represent 50% of the population. As a consequence, the income tax rate that is required to keep the level of debt unchanged increases with the degree of initial human capital equality and with the fraction of the population with access to financial markets. In the single agent case, this rate is

equal to 12.26%, whereas in the $\beta=\gamma=0.5$ case the rate is equal to 5.64%; social security reforms may require big fiscal reforms.[12]

Table 7

Transition Cost as percentage of Output and Capital per unit of

effective labor and Income Tax Rate required for constant Debt for

different values of β and γ

Values of β and γ	Cost/k (%)	Cost/y (%)	τ_{ff}

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[12] In this case, we are assuming that the government had no previously accumulated assets. In most cases, however, countries decide to privatize social security before running out of reserves. This obviously would imply lower tax rates.

β= .5, γ=.5	53.99	5.63	5.64
β=.4 , γ=.7	44.15	5.63	7.63
β=.6, γ=.6	39.18	5.63	8.08
β=.5, γ=.7	38.53	5.63	8.24
β=.5, γ=.8	33.68	5.63	9.63
β=.7 γ=.7	32.17	5.63	10.16
β= .6 , γ=.8	31.56	5.63	10.39
β=.7, γ=.8	30.08	5.63	11.00
β=.5, γ=.9	29.91	5.63	11.07
β= .6 , γ=.9	29.03	5.63	11.48
β=.8, γ=.8	28.95	5.63	11.51
β= .7 , γ=.9	28.43	5.63	11.77
β=.8, γ=.9	28.01	5.63	11.98
β=.9, γ=.9	27.70	5.63	12.14
β=1, γ=1	27.47	5.63	12.26

3. The Transition Path

The previous analysis looked only at the characteristics of the economy at both steady states. It is also interesting to study the behavior of several variables during the transition path from one system to the other. It is also of particular interest to know how long it takes to get from one steady state to the other.

Equation (34') can be used in order to follow the behavior of the economy during the transition path. Using the steady state level of capital per unit of effective labor of the PAYG equilibrium as the initial level of capital and the transition cost as the initial level of debt, we can obtain the level of capital for each successive period after the reform. Table 8 shows the evolution of the wage rate, the interest rate, the level of capital per unit of effective labor and the income tax rate during the transition from a PAYG social security system to a FF one when the government decides to keep the level of debt unchanged. For

simplicity, we make the initial (PAYG) levels of capital, wages and interest rates equal to one.

Table 8

The Economy during the Transition Path:

Homogeneous Agent Case

Period	Capital	Debt	Tax rate	Wage	Int. rate
0	1	0	0	1	1
1	1.176	0.274	11.78%	1.084	0.892
2	1.152	0.274	12.03%	1.073	0.905
3	1.140	0.274	12.15%	1.067	0.912
4	1.134	0.274	12.21%	1.065	0.915
10	1.128	0.274	12.26%	1.062	0.918
Steady State	1.128	0.274	12.26%	1.062	0.918

Table 8a

The Economy during the Transition Path:

$\beta=\gamma=0.8$

Period	Capital	Debt	Tax	Wage	Int. Rate
0	1	0	0	1	1

Period	Capital	Debt	Tax	Wage	Int. Rate
1	1.240	0.289	11.77%	1.113	0.860
2	1.254	0.289	11.64%	1.120	0.853
3	1.261	0.289	11.58%	1.123	0.849
4	1.264	0.289	11.55%	1.124	0.848
10	1.268	0.289	11.53%	1.126	0.846
Steady State	1.268	0.289	11.52%	1.126	0.846

Table 8b

The Economy during the Transition Path:

$$\beta=\gamma=0.6$$

Period	Capital	Debt	Tax	Wage	Int. Rate

0	1	0	0	1	1
1	1.678	0.391	11.76%	1.295	0.695
2	2.038	0.391	9.69%	1.427	0.607
3	2.236	0.391	8.83%	1.495	0.569
4	2.339	0.391	8.45%	1.529	0.551
10	2.442	0.391	8.26%	1.562	0.535
Steady State	2.444	0.391	8.17%	1.563	0.534

As Tables 8, 8a and 8b show, the economy reaches its new steady state ten periods after the reform. Most of the change, however, occurs after 3 periods. At this point, it is important to realize that one period here is the entire working life of an individual, in a more realistic context, it would be equivalent to some 40 to 45 years. These transitions might take a long time. Table 8a shows the transition for the $\beta = \gamma = 0.8$ case. As it was mentioned before, in this case social security reform leads to a new steady state where the stock of capital per unit of effective labor is 26.8% higher. But, as the Table shows, almost 80% of this increase happens by the second period. It should also be noted that, as expected, as the stock of capital grows, the income tax rate decreases. Also, the wage rate increases as the extra capital makes workers more productive; at the same time the interest rate falls. In this case, interest rate falls by 34.1% and wages increase by 12.6% after the transition is over.

The single agent case is somewhat different. As Table 8 shows, capital per unit of effective labor increases by 17.6% the period after the reform. Then the high debt to capital ratio crowds out capital the following periods until the economy reaches the new steady state where the stock of capital is higher than in the Pay-as-you-go case but lower that in the first period after the reform. In this case, the income tax rate increases with time: future generations pay a higher portion of the transition cost. The Table also shows that the wage rate increases by 6.02% and the interest rate decreases by 16%.

The 2 period model may underestimate the cost of the transition because it implies that the government only has debt with one generation. In the annex we present an extension to the model with more realistic transition paths.

4. Conclusion

This paper presented an heterogeneous agent model developed to study the transition from a state-managed Pay-as-you-go social security system to a privately-managed Fully-funded one. We assumed that agents can differ in their human capital endowments and in their access to the financial system. We find that, for some initial distributions, when access to the financial system is restricted for some individuals, income distribution may improve with the privatization of the pension system. In the case in which there is complete access to the financial system before the reform, however, income distribution deteriorates in all cases.

Regardless of the initial distributions, a reform of the type described here increases the level of physical capital in the economy. However, the increase will be larger the larger are the fraction of the population composed by poor individuals or the higher their level of human capital.

We also find that different initial distributions will have different effects on the fiscal policy needed to finance the reform. Similarly, different forms of financing the reform will have different effects on intragenerational distribution. In the case in which the government decides to maintain a constant level of debt, generations alive when the reform takes place will have lower lifetime earnings than those born after them. We also find that the taxes needed to pay for transitional workers' pensions will be higher when the fraction of population with access to the financial system in the PAYG equilibrium is higher.

This paper illustrates how social security reforms in countries with underdeveloped financial systems can affect income distribution, fiscal policy and growth.

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ANNEX
An Extended Version of the Model

In this Annex, following Auerbach and Kotlikoff (1987), we introduce an extended version of the model presented in the paper. The extension mainly consists in introducing realistic life span periods. Specifically, individuals will live for 55 periods instead of 2. This life span is realistic since the model assumes that individuals start working immediately after they are born; this equivalent to think that individuals start working at age 21, work for several years of their lives, retire and die at age 75. [13] This implies that, at any given period in time, there will be 55 different generations alive. We still hold the assumption of 2 types of individuals in each generation, and that of poor agents not having access to the financial system. This extended version of the model will provide a more clear idea of the development of the economy during the transition path from one system to the other. Also, it will allow us to put together the elements discussed above - fiscal policy, income distribution, etc.   In addition, introducing more realistic lifetime spans some insights are gained, for example, as Auerbach and Kotlikoff point out: "interest rate changes may alter the present value of lifetime labor earnings."

Individuals of both types will maximize their lifetime utility. Again, we use logarithmic utility functions. Lifetime utility functions are given by:

$$U^i = \sum_{t=1}^{55} \left(\frac{1}{(1+r)}\right)^{t-1} \ln C_t^i \qquad\qquad \text{i=1,2} \qquad\qquad \textbf{(36)}$$

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[13] We use this value because it is equal to life expectancy in Mexico.

That is, each individual lives for 55 periods and has a constant discount factor during his entire life[14]. Individuals of type 1, (rich individuals) face the following budget constraint:

$$a_{t+1}^1 = (1+r_t)a_t^1 + (1-t_s)h^1w_t + p_t - t(r_ta_t^1 + h^1w_t) - c_t^1 = 0 \qquad \textbf{(37)}$$

where a_t^i are financial assets held by an individual of type i at period t. p_t is the retirement pension received by the individual at time t. It should be noted that the amount of pension payment does not depend on the individual's type; again this reflects the redistributive character of the system: individuals are taxed according to their labor incomes and receive an equal pension at retirement. The remainder variables are the same as in the previous section.[15]

And the budget constraint faced by individuals of type 2, when the PAYG system prevails, is given by:

$$a_{t+1}^2 = a_t^2 + (1-t_s)h^2w_t + p_t - th^2w_t - c_t^2 = 0 \qquad \textbf{(38)}$$

Again, it should be noticed that these individuals do not receive interest payments on their savings. All agents retire at age R. Each person will receive wages only during his working years and pension payments only during his retirement years:

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[14] We assume that both types have the same utility function in order to show in a more clear way the effect of poor agents to the having an important demand for savings but no access to the financial system.

[15] Notice that the endowment of human capital is now denoted with lower- case letters rather than with upper-case. This is because now there will be several working-age generations living at the same time, and we have to differentiate each of them with the total stock of human capital.

$$h_t^i = 0 \qquad \text{for } t = R+1, ..., T.$$

$$p_t = 0 \qquad \text{for } t = 1, ..., R.$$

Also, since individuals only care about their own well being, there are no bequests or inheritances so: $a_t(0) = a_t(56) = 0$. Where $a_t(j)$ are assets held by an individual of age j at time t. The first order condition for individuals of type 1 is:

$$\frac{C_{t+1}}{C_t} = \frac{1 + r_{t+1}}{1+r} \qquad \text{(39)}$$

and the equivalent expression for agents of type 2 is given by:

$$\frac{C_{t+1}}{C_t} = \frac{1}{1+r} \qquad \text{(40)}$$

Since these agents do not receive interest payments, their consumption path does not depend on the interest rate. Production is still of the Cobb-Douglas form and payments to factors are given by equations (17) and (18). Now, the total stock of human capital is given by the sum of the endowments of each generation alive:

$$H^1 = \sum_{j=0}^{55} h^1(j) \qquad \text{(41)}$$

$$H^2 = \sum_{j=0}^{55} h^2(j) \qquad \text{(42)}$$

$$H = H^1 + H^2 \qquad (43)$$

Where $h^i(j)$ is the stock of human capital held by agents of type 1 and age j at any period. Agents of type 1 are still assumed to hold a fraction γ of the stock of human capital and a fraction β of the labor force. Similarly, total labor force is the sum of generation size across all living generations.

$$L_t^1 = \sum_{j=0}^{55} l_t^1(j) \qquad (44)$$

$$L_t^2 = \sum_{j=0}^{55} l_t^2(j) \qquad (45)$$

where $l_t^i(j)$ is the number of persons of age j and type i alive at period t.

$$L_t = L_t^1 + L_t^2 \qquad (46)$$

Population is still assumed to grow at rate n:

$$L_t = (1+n)L_{t-1} \qquad (47)$$

Aggregation of financial assets is as follows:

$$A_t^1 = \sum_{j=0}^{55} l_0^1(1)(1+n)^{t-j+1} a_t^1(j) \qquad (48)$$

$$A_t^2 = \sum_{j=0}^{55} l_0^2(1)(1+n)^{t-j+1} a_t^2(j) \tag{49}$$

$$A_t = A_t^1 + A_t^2 \tag{50}$$

And the aggregate level of pension payments is the sum of individual pensions across generations:

$$P_t = \sum_{j=0}^{55} [l_0^1(1) + l_0^2(1)](1+n)^{t-j+1} p_t(j) \tag{51}$$

A.1 Introducing Government

We still assume that government is only concerned with the social security system and faces the same budget constraint as in the previous version:

$$B_{t+1} = B_t(1+r_t) + G_t - [t_s w_t (H^1 L_t^1 + H^2 L_t^2) + t_t Y_t] \tag{52}$$

As in the two period version, government expenditures G, will be different from zero only during the transition from the PAYG to the FF system and they will be equal to pensions paid to transitional workers. We assume that the level of debt is zero before the reform.

A.2 The transition and the steady states under both systems

As we already explained, in the PAYG system the government uses contributions from current workers to pay retirement pensions that are equal for all individuals. This implies that:

$$p_t = \frac{P_t}{\sum\limits_{j=R}^{55} [l_t^1(j) + l_t^2(j)]} \tag{53}$$

and:

$$P_t = t_s w_t (H^1 L_t^1 + H^2 L_t^2) \tag{54}$$

Therefore, in the PAYG steady state $B_t = B_{t+1} = 0$. The stock of capital in the PAYG equilibrium is:

$$K_t = A_t^1 - B_t \tag{55}$$

Therefore, the equation that describes the evolution of the capital stock in the PAYG steady state is:

$$K_{t+1} = A_{t+1}^1 - A_t^1 + K_t \tag{56}$$

A.3 The Fully- Funded system

Recall that the adoption of a FF system means that the government will receive no more contributions, but at the end of the transition, it will no longer be responsible of paying retirement pensions. At the same time, retirement pensions will now depend directly on the contributions made to the social security during the working years. This implies that pension amounts will be different for our two types of individuals. Specifically, pensions received by an individual of type i are given by:

$$p_t^i(j) = \frac{\sum\limits_{s=0}^{R}(1+r_s)^{R-s+1} t_s h^i w_s}{55-R} \cdot \prod_{s=R}^{54}(1+r_s)^{s-R} \quad \text{for j>R} \qquad \textbf{(57)}$$

That is, the balance accumulated in the individual accounts during the working years (contributions plus interest payments) will be equal to the value of the pensions received during the retirement years; there is no intra-generation redistribution. Also, we assume that this value will be divided across the retirement years so that each pension will have the same value except for the interest that is paid on the remaining balance of the retirement account.

Recall our assumption that the reform will give access to the financial system to agents of type 2; their budget constraint in the FF system is now:

$$a_{t+1}^2 = (1+r_t)a_t^2 + (1-t_s)h^2 w_t + p_t - t(r_t a_t^2 + h^2 w_t) - c_t^2 = 0 \qquad \textbf{(58)}$$

and the equilibrium condition is:

$$K_{t+1} = A_{t+1} - A_t + K_t \qquad \textbf{(59)}$$

A.4 The Transition from the PAYG to the FF system

In order to obtain solutions for this model, we assign realistic values for the parameters[16] and perform computer simulations using the Gauss-Seidel method[17]. This method consists in using initial guesses for the vectors of factor prices, performing iterations plugging these guesses into the first order conditions and, with the consumption paths implied by these conditions,

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[16] We use the same parameter values as in the two period version and we assume that retirement age R is 45 years.

[17] The simulations were performed using Gauss software.

obtaining aggregate levels of capital and effective labor. The iterations continue until the factor prices consistent with these aggregate levels of capital and labor are equal to the initial guesses. For a complete description of this solution method see Auerbach and Kotlikoff (1987, chapter 4.)

We simulate the transition from a PAYG to a FF system using alternative forms of financing the transitional cost previously described. Again we study the effects of the reform on fiscal policy, capital accumulation and income distribution; in this section, in addition to distribution within generations, we discuss distribution between generations.

We consider the case in which the fiscal policy consists in imposing an income tax such that the level of debt is constant during the transition. In particular, we assume that the level of debt is kept equal to zero. Figure 3 shows the evolution of physical capital along the transition between both systems for three different combinations of β and γ . In the homogeneous agent case, the steady state level of capital in the FF system is 14.83% higher than in the PAYG one; a result that is consistent with the studies previously cited. Note that initially there is a decrease in the stock of capital: the taxes needed to maintain a zero debt path have a crowding out effect on capital. Twenty one years after the transition, the stock of physical capital reaches its minimum level: 0.25% lower than the PAYG steady state value. It can also be seen that, as in the two period model, the higher the fraction of the stock of units of effective labor in hands of poor agents, the lower the steady state level of physical capital in the PAYG steady state and, therefore, the higher its increase after the reform . In the $\beta= 0.6$, $\gamma =0.8$ case, for example, capital in the FF equilibrium is 17.90% higher than in the PAYG one.

Figure 4 shows changes in intergenerational distribution. The vertical axis depicts the percentage difference on lifetime wealth for generations born at

different years with respect to the generation born 60 years before the reform.[18] We can see that those generations born 55 or less years after the reform takes place, that is generations that are alive by then, will see a decrease in their welfare. (Generations born before that will not be affected at all.) These generations are worse off compared to those born after the reform because they do not perceive the benefits of a higher stock of capital and, therefore, of a higher consumption path. Those that are born just before the reform lose the most because they have to continue with the old system and, at the same time, they have to share (by paying income taxes) the reform's cost. Generations born after the reform begins will be better off. In the homogeneous agent case, lifetime wealth of generations born in the new FF steady state (about 90 years after the reform) will be 6.28% higher than that of generations born 55 or more years before the transition. On the other hand those generations born immediately after the reform perceive a 2.88% increase in their lifetime wealth.

It can also be seen that all generations, regardless of their year of birth, will have higher welfare as the fraction of units of effective labor in hands of agents of type 1 is lower. For example, in the $\beta = 0.4$, $\gamma = 0.7$ case, generations born immediately after the reform have a lifetime wealth that is 9.33% higher than their equivalent generations in the homogeneous agent case. Similarly, individuals born before the reform, in cases where β and γ are lower than 1, do not lose as much as in the homogeneous agent case.   This result supports our conclusion that the benefits of adopting a FF system will be higher as the fraction of the population that does not have access to the financial system in the PAYG system is higher.

Figures 5 and 6 show the evolution wages and interest rates during the transition. The higher levels of physical mean higher wages and lower interest rates. As expected, wages increase more and interest rate fall more for lower values of β and γ.

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[18] Here we measure welfare of the entire generation.

Figure 7 shows the changes in the income tax rate during the transition. This rate increases as the generations that still have to receive pensions paid by the government retire. After the last generation that will be retired under the old scheme retires (45 years) the income tax rate falls as the old scheme's pensioners die. As in the two period version, the income tax rate needed to maintain a constant level of debt is higher with higher values of β and γ: in the homogeneous agent case the income tax rate is, at its peak, 7.97%, whereas in the $\beta = 0.4$, $\gamma = 0.7$ case this rate 5.56%. In all cases, the income tax rate reaches its maximum 46 years after the reform . Finally, Figure 8, shows the transition cost as percentage of GDP (assuming that all transitional workers retire under the old system) for the homogeneous agent case. This cost remains steady for the first 45 years since, in that period, each year one new generation starts receiving pensions and one generation dies. After that, the cost starts to decrease as no one else retires under the old system and as existing pensioners die.
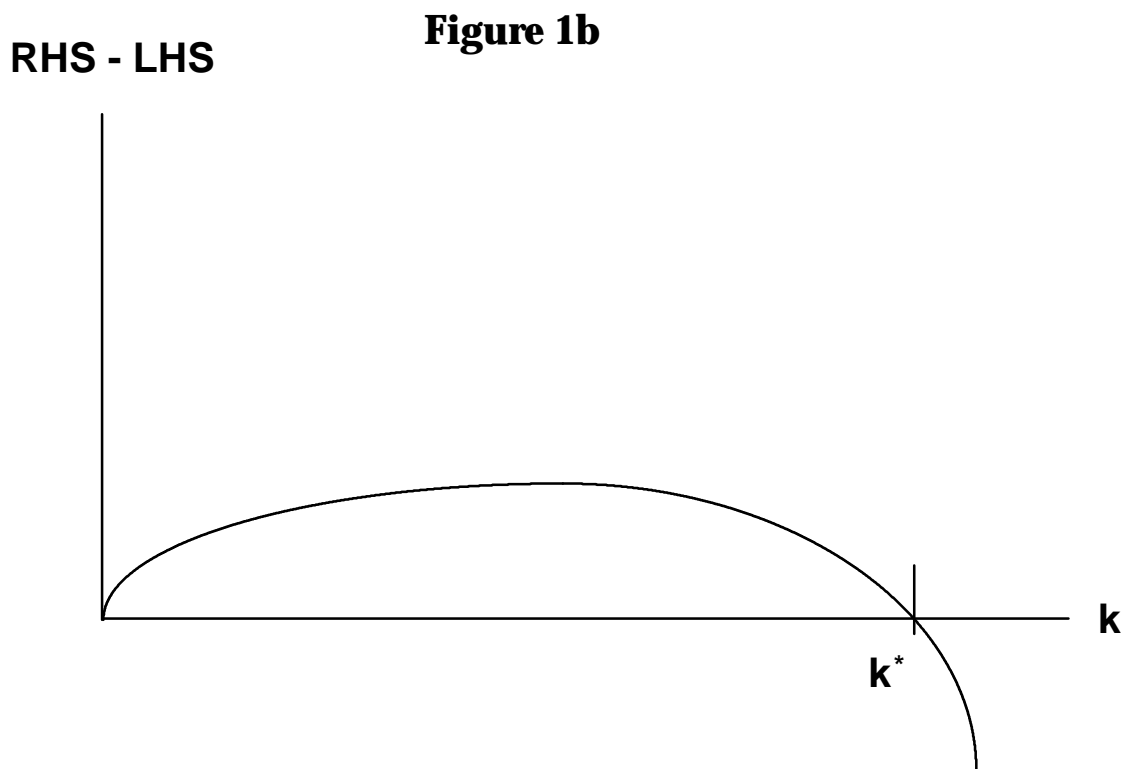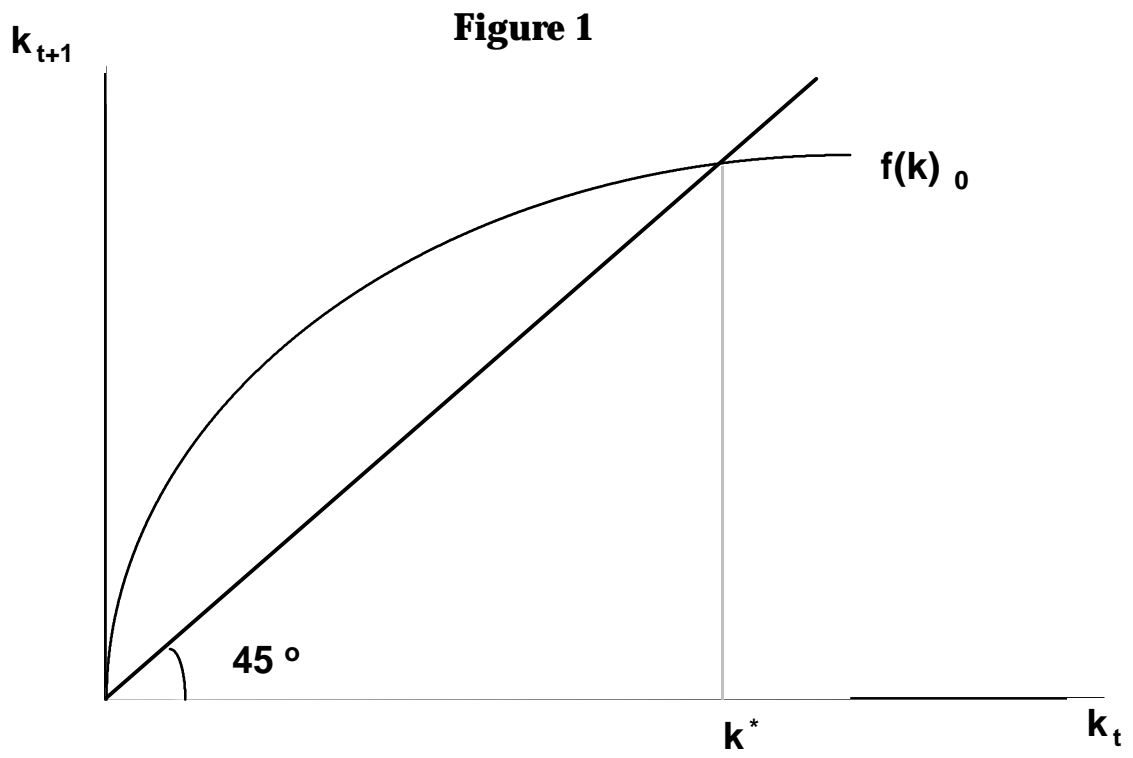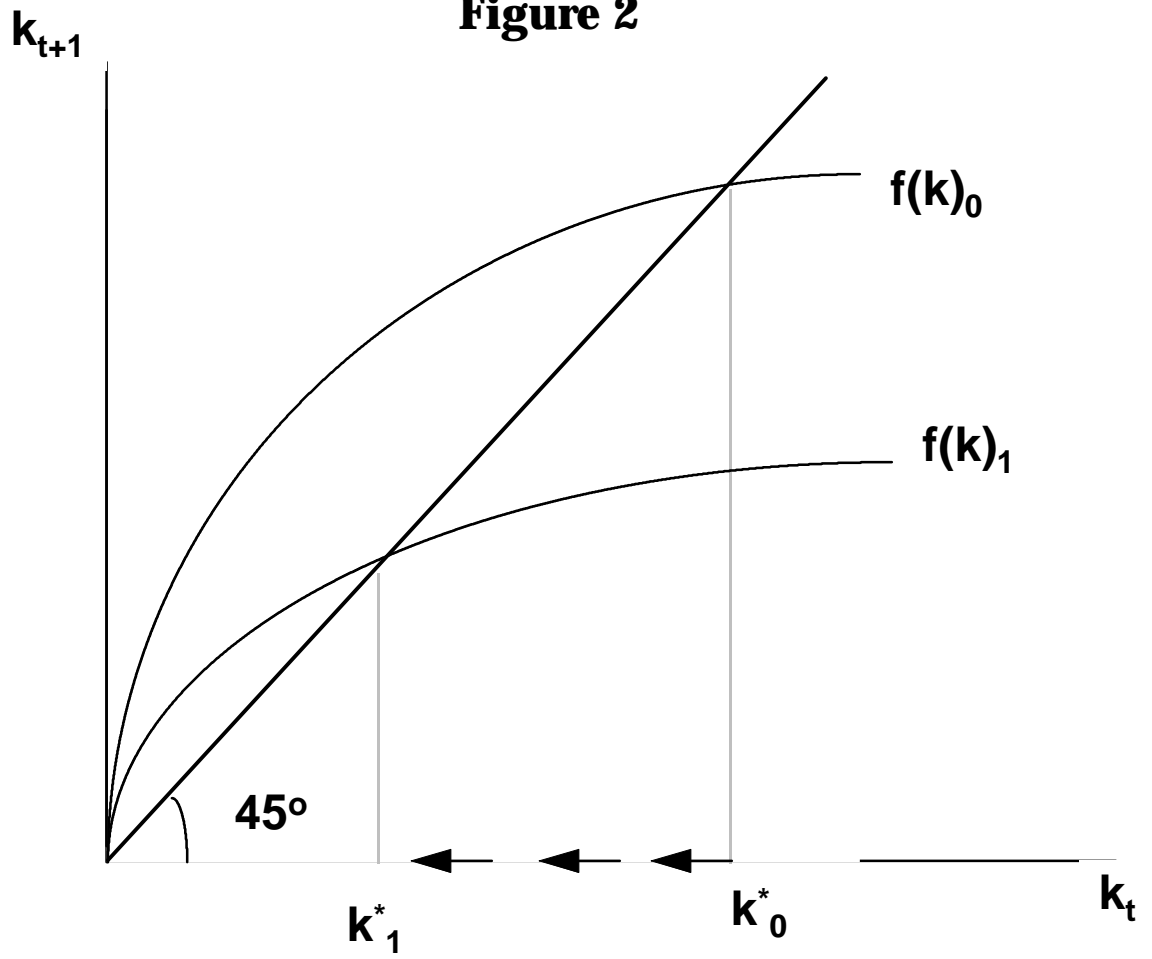
Figure 1

k_{t+1}

$f(k)_0$

45 °

k^*

k_t

Figure 1b

RHS - LHS

k

k^*

Figure 2

FIGURE 3

CAPITAL EVOLUTION FOR DIFFERENT VALUES OF BETA

AND GAMMA

FIGURE 4

INTERGENERATIONAL WELFARE FOR DIFFERENT VALUES

OF BETA AND GAMMA

FIGURE 5

WAGE EVOLUTION FOR DIFFERENT VALUES OF BETA AND GAMMA

FIGURE 6

INTEREST RATE EVOLUTION FOR DIFFERENT VALUES OF

BETA AND GAMMA

FIGURE 7

INCOME TAX RATES FOR DIFFERENT VALUES OF BETA AND GAMMA

FIGURE 8

FISCAL COST OF THE TRANSITION (AS % OF GDP)